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Active Control of Vibration and Noise

OPTIMAL CONTROL OF TURBULENT CHANNEL FLOWS

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ABSTRACT

Feedback control equations are developed and tested using optimal control theory for two different flow control problems. In the first, wall-normal blowing and suction distributions are computed to efficiently reduce drag. The technique used to compute the control is the minimization of a "cost functional" which is constructed to represent some balance of the drag integrated over the walls and the net control effort. Minimization of this cost functional is achieved by utilizing an adjoint problem to determine the sensitivity of the flow to control, then updating the control with a gradient algorithm. A distribution of wall velocities is thus found which minimizes the cost functional over a short time interval based on current observations of the flow near the wall.

Preliminary numerical simulations of this scheme applied to turbulent channel flow indicates it provides approximately 17% drag reduction with small levels of control input. The mechanism apparent when the scheme is applied to a simplified flow is also discussed.

The second problem considered is the computation of efficient internal forcing distributions to reduce pressure fluctuations on the walls of a channel. The internal forcing simulated is of a type which might be produced by appropriately placed magnets and electrodes in a wall acting on near-wall fluid, referred to in recent literature as Electro-Magnetic Turbulence Control (EMTC). The optimal formulation for this problem is similar to that in the drag reduction by wall transpiration problem. Computational results of this scheme, which were not available at the time of printing, will be presented at the conference.

1. INTRODUCTION

Wall-bounded turbulent flows are dominated in the near-wall region by longitudinal vortex structures (Robinson 1991). These vortex structures create inrushes of high momentum fluid toward the wall, called sweep events, and outward movement of low momentum fluid near the wall back into the center of the channel, called ejection events. Such phenomena result in several effects, including increased drag, increased heat transfer to or from the wall, and wall-pressure fluctuations which generate sound (which emanates into the fluid) and structural vibrations. Depending on the particular problem under consideration, these effects may be beneficial or detrimental, and the control engineer is motivated to investigate methods to modify them. Wall transpiration and EMTC forcing of near-wall fluid are considered in this work as possible methods to modify these near wall-structures in an active feedback configuration.

Section 2 will introduce the optimal control method with a strategy for drag reduction by wall-transpiration. The method developed there is described in Abergel and Temam (1990) in a related situation and is also discussed in Lions (1969). Section 3 will present preliminary results of this scheme applied to a direct numerical simulation of turbulent channel flow. Section 4 will discuss how the optimal control strategy may be extended to the problem of reduction of wall-pressure fluctuations by EMTC forcing.

In Sections 2 and 3, a small amount of wall-normal blowing and suction is used to apply the control. With a well-chosen scheme using wall transpiration only, it has been shown that a turbulent flow may be smoothed out in a near-wall region, and the drag may be substantially reduced; for example, the *ad hoc* schemes of Choi *et al.* (1994) reduced turbulent drag by as much as 20% by countering the vertical velocity slightly above the wall with an equal but opposite control velocity at the wall. The present work demonstrates how possibly more efficient schemes may be derived by applying optimal control theory, utilizing the equations of motion of the fluid to reveal the dominant physics of the control problem.

The forcing technique investigated in Section 4 has received growing attention over the last few years due to some interesting experiments with EMTC for the purpose of turbulent drag reduction carried out at Princeton (Nosenchuck and Brown, 1993). The forcing profile used in the current work is a rough model of what might be feasible using electromagnets and an array of electrodes mounted on the wall which induce a Lorenz force on an electrolytic fluid (like seawater) flowing above the wall. In the present work, this control technique is used in an active feedback configuration to interact with turbulent structures directly.

The model problem we shall study is the turbulent flow inside a small segment of a fully developed turbulent channel (*i.e.* flow between two parallel walls, far from the inlet), with periodic boundary conditions used in the spanwise and streamwise directions and a constant mass flux maintained by variation of the external pressure gradient. This flow is governed by the same vortical structures as a turbulent boundary layer flow in the near-wall region, and is much less expensive to simulate. The domain is chosen to be large enough that the non-physical periodic boundary conditions do not affect the nature of the turbulence (Kim *et al.* 1987).

For practical implementation, the solution to the optimal control problems may be approximated by schemes which depend only on information which may be measured at the wall with an array of flush-mounted sensors. These approximate schemes lead to adjoint problems which, through further approximations, may be solved analytically. Such analytic approximations lead to simple transfer functions which relate the measurements made by sensors in an experimental implementation to the inputs of the actuators (Hill 1993). Methods of implementing active turbulence control ideas, including a discussion of several sensors and actuators appropriate for such problems, have recently been reviewed by Moin and Bewley (1994).

2. DRAG REDUCTION BY WALL TRANSPIRATION—FORMULATION

The problem under consideration in this section is a turbulent channel flow with no-slip walls and wall-normal control velocities Φ. This problem is governed by the unsteady, incompressible Navier-Stokes equation and the continuity equation inside the domain Ω and appropriate boundary conditions on the walls w (periodic conditions are implied on the remainder of the boundary of the domain Γ):

$$\begin{aligned} \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} - \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_j^2} + \frac{1}{\rho}\frac{\partial p}{\partial x_i} &= 0 \\ \frac{\partial u_j}{\partial x_j} &= 0, \end{aligned} \qquad (1a)$$

with boundary conditions

$$u_i = \Phi\, n_i \qquad \text{on walls} \qquad (1b)$$

and prescribed initial conditions

$$u_i = u_i(t_0) \qquad \text{at } t = t_0, \qquad (1c)$$

where x_1 is the streamwise direction, x_2 is the wall-normal direction, x_3 is the spanwise direction, u_i are the corresponding velocities, p is the pressure, ρ is the density, Re is the Reynolds number, δ is the channel half-width, and n is a wall-normal unit vector directed *into* the channel, as illustrated in Figure 1.

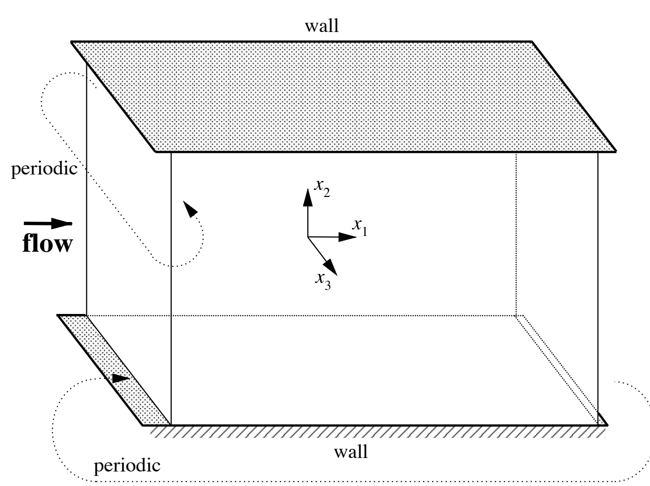

FIGURE 1. Flow configuration. Blowing and suction is applied through holes drilled in the walls to control the flow.

As mentioned in the abstract, the first step in solving an optimal control problem is to represent the control problem of interest as a cost functional, $\mathcal{J}$, to be minimized. In the present problem, control is to be applied to minimize the drag on a section of wall with area A over a period of time T using the least amount of control effort possible. A relevant cost functional the present problem is thus

$$\mathcal{J}_d(\Phi) = \frac{\ell}{2\,A\,T}\int_w \int_{t_0}^{t_0+T} \Phi^2\, dt\, dS + \frac{1}{A\,T}\int_w \int_{t_0}^{t_0+T} n_2 \frac{\partial u_1}{\partial x_2}\, dt\, dS. \tag{2}$$

The first term on the right hand side is a measure of the magnitude of the control. The second term is a measure of exactly that quantity we would like to reduce—in this case, the drag. (Note the factor n_2 needed to account for the orientation of the upper wall.) These quantities are integrated over the wall sections under consideration, of area A, and over the time period under consideration, beginning at $t = t_0$ and of duration T. Finally, they are weighted together with a factor ℓ, which represents the price of the control. This quantity is small if the control is "cheap" (which reduces the significance of the first term on the right hand side and, in general, results in larger control velocities), and large if applying control is "expensive". The form of the cost functional is similar for other flow control problems, as seen in Section 4. Note that the integrand of the second term on the right hand side is not a non-negative quantity; it has been observed in the course of these investigations that the most effective cost functional for a particular purpose incorporates exactly that term which one would like minimized.

As derived in Abergel and Temam (1990) and investigated further by Choi *et al.* (1993), a procedure may be developed using adjoint calculus to determine the sensitivity of the cost functional $\mathcal{J}_d$ to the control Φ. To do this, we define a <u>flow state</u>

$$U = \begin{pmatrix} u_i(x_1, x_2, x_3, t) \\ p(x_1, x_2, x_3, t) \end{pmatrix}$$

which is governed by (1), a <u>differential state</u>

$$\dot{U} = \begin{pmatrix} \dot{u}_i(x_1, x_2, x_3, t) \\ \dot{p}(x_1, x_2, x_3, t) \end{pmatrix},$$

where $\dot{U}$ is defined using a Fréchet differential (Vainberg, 1964) such that

$$\dot{U} \equiv \frac{\mathscr{D}U(\Phi)}{\mathscr{D}\Phi}\tilde{\Phi} = \lim_{\epsilon\to 0}\frac{U(\Phi+\epsilon\tilde{\Phi}) - U(\Phi)}{\epsilon},$$

which is governed by the Fréchet differential of (1), and an <u>adjoint state</u>

$$\tilde{U} = \begin{pmatrix} \tilde{u}_i(x_1, x_2, x_3, t) \\ \tilde{p}(x_1, x_2, x_3, t) \end{pmatrix}$$

which is *defined* by

$$\left.\begin{aligned} -\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial u_j}{\partial x_i} - u_j \frac{\partial \tilde{u}_i}{\partial x_j} - \frac{1}{Re}\frac{\partial^2 \tilde{u}_i}{\partial x_j^2} + \frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_i} &= 0 \\ \frac{\partial \tilde{u}_j}{\partial x_j} &= 0 \end{aligned}\right. \tag{3a}$$

with boundary conditions

$$\tilde{u}_i = \begin{cases} 1 & i = 1 \\ 0 & \text{otherwise} \end{cases} \qquad \text{on walls} \tag{3b}$$

and initial conditions

$$\tilde{u}_i = 0 \qquad \text{at } t = t_0 + T. \tag{3c}$$

The equation governing $\dot{U}$, which is found by taking the Fréchet differential of (1a), is linear in the variable $\dot{U}$ and may thus be written

$$A\dot{U} = 0. \tag{4a}$$

The equation governing $\tilde{U}$ in (3a) is linear in $\tilde{U}$ and may be written

$$A^*\tilde{U} = 0. \tag{4b}$$

The equation used to define the adjoint operator A^* is

$$< A\dot{U}, \tilde{U} > = < \dot{U}, A^*\tilde{U} > + b \tag{5}$$

where the inner product is defined

$$< \dot{U}, \tilde{U} > = \int_\Omega \int_{t_0}^{t_0+T} \dot{U}\cdot\tilde{U}\, dt\, dV.$$

Equation (5) may be simplified using (4a), (4b), and the boundary and initial conditions on $\dot{U}$ and $\tilde{U}$ such that only a two of the boundary terms in b remain. The resulting expression may be written as

$$\frac{1}{A\,T}\int_w \int_{t_0}^{t_0+T} n_2 \frac{\partial \dot{u}_1}{\partial x_2}\, dt\, dS = -\frac{Re}{\rho\, A\, T}\int_w \int_{t_0}^{t_0+T} \tilde{p}\,\tilde{\Phi}\, dt\, dS.$$

The Fréchet differential of the cost functional $\mathcal{J}_d$ in (2) may be rewritten using this expression and the gradient extracted, which results in

$$\frac{\mathscr{D}\mathcal{J}_d(\Phi)}{\mathscr{D}\Phi} = \frac{\ell}{A\,T}\Phi - \frac{Re}{\rho\, A\, T}\tilde{p}.$$

Thus, the sensitivity of the cost functional to control may be determined using the solution to an adjoint problem. A feedback control strategy using a simple gradient algorithm may now be proposed such that

$$\Phi^k = \Phi^{k-1} - \mu\,\frac{\mathscr{D}\mathcal{J}_d(\Phi^{k-1})}{\mathscr{D}\Phi}, \tag{6}$$

where k indicates the iteration step for the time interval $t \in (t_0, t_0 + T]$ and μ is a parameter of descent which governs how large an update is made at each iteration. This algorithm attempts to update Φ in the direction of maximum decrease of $\mathcal{J}_d$. For small μ as $k \to \infty$, the algorithm should converge to some local minimum of $\mathcal{J}_d$ over the control space Φ. Note that convergence to a global minimum will not necessarily be attained.

To make the optimal control method practical for implementation, we consider only very short time durations T when considering the influence of the control; we shall call this the suboptimal approximation. With this approximation, we may "freeze" the flow field when computing the adjoint, and the adjoint may be determined in a single computational time step. These steps simplify the calculation of the control update significantly. Without this approximation, an optimal scheme has very large storage requirements. This is due to the fact that the development of U over the entire time interval T is required by (3) to compute the adjoint.

By considering only small values of T, the control algorithm gives the control which minimizes the cost functional over some short time interval with reduced computational and storage requirements. Note, however, that this method does not look ahead to anticipate further development of the flow, and thus the solution by this method does not necessarily correspond to the solution by the optimal control method. Thus, posing the problem in this suboptimal form is an approximation to the physical problem of interest. The accuracy of this approximation, which may be determined only by careful study of the results of both optimal and suboptimal control schemes, remains to be established.

3. DRAG REDUCTION BY WALL TRANSPIRATION—RESULTS

Elementary drag reducing mechanisms

Choi *et al.* (1994) found that, by applying a control velocity equal and opposite to the vertical velocity at $y^+ = 10$, a drag reduction of about 20% could be achieved. Vertical transport of streamwise momentum in the near-wall region (primarily due to longitudinal vorticity) produces "sweep" events and thus local regions of very high drag. Applying a countering control velocity tends to reduce this effect.

In the transverse plane, countering the vertical velocity above the wall corresponds to the control sketched in Figure 2. This type of control corresponds to blowing where the drag is high, which decreases the high velocity gradients at the wall and thus smoothes out the flow in the near-wall region, as shown in Figure 3.

Figure 4 shows the application of the suboptimal control scheme to a simple flow configuration of longitudinal vortices embedded in an initially parabolic flow. A cross flow plane is shown. In regions below downward moving fluid (sweep events) the streamwise (into the page) drag is higher and blowing is applied. In regions below upward moving fluid (ejection events), the streamwise drag is lower and suction is applied.

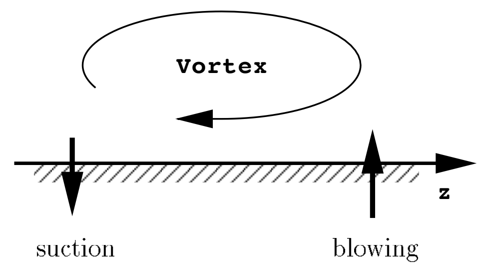

FIGURE 2. Stabilization mechanism in cross flow plane.

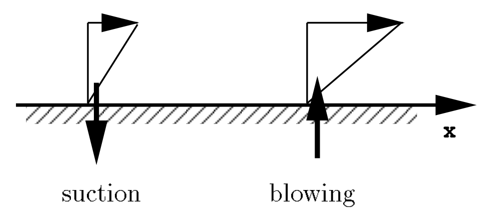

FIGURE 3. Stabilization mechanism in an x-y plane. High drag is decreased by blowing at the expense of suction in the regions of low drag, resulting in a net smoothing of the near-wall velocity profiles.

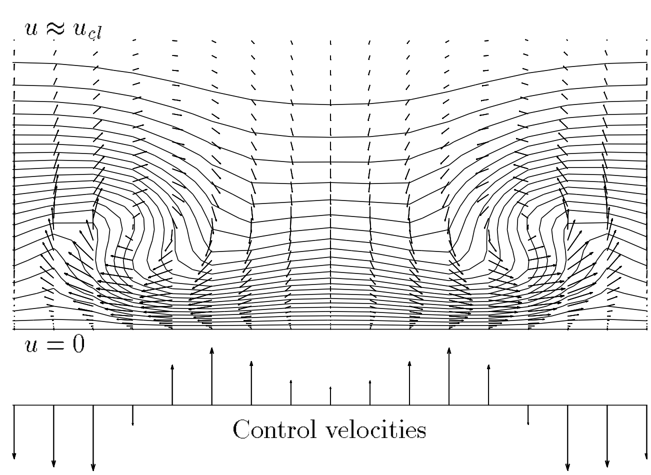

FIGURE 4. Suboptimal control scheme applied to longitudinal vortices. Interior vectors are cross flow velocities and contours are of streamwise velocity, indicating a sweep event between two near-wall vortices and ejection events outside of them. Control velocities shown on the wall (not to scale) indicate blowing at the sweep event and suction at the ejection events.

The suboptimal scheme derived in Section 2 was tested by applying it to a direct numerical simulation of turbulent channel flow. A 17% drag reduction was seen as compared to a flow with no control. Results are plotted in Figure 5. This calculation was done in a flow with $Re = 1850$ based on the mean centerline velocity and the channel half width using the spectral method of Kim *et al.* 1987.

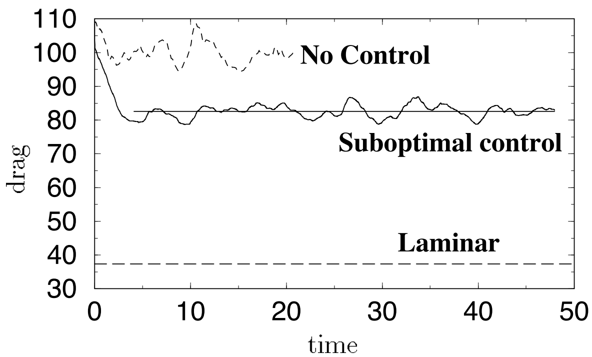

FIGURE 5. Drag reduction of suboptimal scheme compared to the no-control and relaminarized cases. Drag is normalized by percent of mean drag in the no-control case. Time is in units of δ/u_τ in the no-control case. Parameters for suboptimal scheme are $\mu = 0.01$, $\ell = 10$, $T = 1$ wall unit.

The adjoint analysis utilizes all the information present in the channel to extract the sensitivity of the instantaneous drag to the variation of the control. However, only information in a thin layer near the wall is significant in computing the adjoint for the suboptimal problem. Thus, the suboptimal scheme may be reduced to an approximate problem relying only on wall information by approximating the near-wall velocities using a Taylor's series extrapolation of the velocity gradients at the wall. This approximation may be solved analytically, resulting in simple control laws which depend only on information which may actually be measured experimentally at the wall. Such practical schemes are discussed further by Hill (1993).

4. WALL-PRESSURE FLUCTUATION REDUCTION BY EMTC—FORMULATION

The problem under consideration in this section is a turbulent channel flow with solid, no-slip walls and internal control forcing. This problem is governed by the forced Navier-Stokes equation and the continuity equation inside the domain Ω and homogeneous boundary conditions on the walls w:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} - \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_j^2} + \frac{1}{\rho}\frac{\partial p}{\partial x_i} = f_i(\Phi) \qquad \frac{\partial u_j}{\partial x_j} = 0, \tag{7a}$$

with boundary conditions

$$u_i = 0 \qquad \text{on walls} \tag{7b}$$

and prescribed initial conditions

$$u_i = u_i(t_0) \qquad \text{at } t = t_0, \tag{7c}$$

where $f_i(\Phi)$ is the internal control forcing per unit volume, which is a function of the wall potential distribution Φ. We shall consider a configuration similar to that in Section 2, as illustrated in Figure 6.

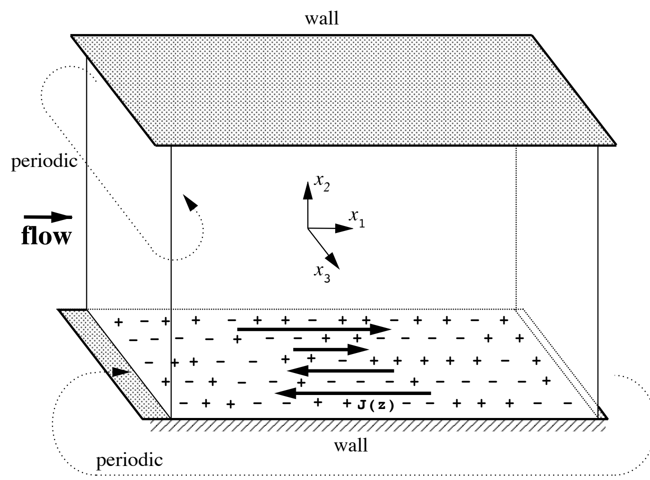

FIGURE 6. Flow configuration. Spanwise-varying wall currents (indicated by the arrows on the walls) provide a steady magnetic field B_i in the channel, and electrodes (indicated by the $+/-$ symbols) provide an unsteady current distribution j_i in the channel.

The control forcing term $f_i(\Phi)$ is a Lorenz force due to electric and magnetic fields acting on the fluid. Electrodes are distributed continuously on the lower wall (insulated from each other but not from the flow), providing an arbitrary potential distribution on this wall. The surface of the upper wall is coated with a grounded conductor. (Control may also be applied at the upper wall, as the electric field may be determined by superposition, but for simplicity of notation we will consider control of only the lower wall.) The channel is also subjected to a magnetic field in the cross-flow plane, which is created by spanwise-varying currents through streamwise-oriented wires placed beneath the array of electrodes on the lower wall, as shown in Figure 6. The steady magnetic field for a particular realization of this configuration may easily be computed, and will be denoted B_i.

The fluid is assumed to be only weakly conducting, like seawater. With this assumption, we can neglect the dynamics of the fluid and the small electric currents when computing the influence of the wall charge distribution on the electric field inside the channel, and may thus compute the electric potential ϕ inside the channel using Laplace's equation:

$$\frac{\partial^2 \phi}{\partial x_j^2} = 0 \qquad (\nabla^2 \phi = 0).$$

The boundary condition at the lower wall is a potential Φ, which is provided by applying a voltage distribution to the electrodes on the lower wall, and the boundary condition at the upper wall is homogeneous:

$$\begin{aligned} \phi &= \Phi \qquad \text{at } x_2 = 0 \\ \phi &= 0 \qquad \text{at } x_2 = 2\delta. \end{aligned}$$

This equation may, of course, be solved analytically by considering individual Fourier modes. For this purpose, the hat notation is used to indicate the Fourier transform in the x_1 and x_3 directions. The resulting potential distribution in the channel is easily found:

$$\hat{\phi} = \hat{g}\,\hat{\Phi} \tag{8}$$

where $\hat{g} = (e^{-x_2 k} - \beta e^{x_2 k})/(1-\beta)$, $k = \sqrt{k_1^2 + k_3^2}$, and $\beta = e^{-4\delta\, k}$.

A conduction current with density j_i will flow in a direction parallel to the electric field, which may be found by taking the gradient of the potential field, in proportion to the conductivity σ of the fluid:

$$j_i = \sigma \frac{\partial \phi}{\partial x_i} \qquad (\mathbf{j} = \sigma \nabla \phi). \tag{9}$$

For seawater at an average salinity, $\sigma \approx 4$ (ohm m)$^{-1}$. As the current density j_i is not coincident with the magnetic field B_i, a Lorenz force is exerted on the fluid in proportion to their cross product:

$$f_i = \epsilon_{imn}\, j_m\, B_n \qquad (\mathbf{f} = \mathbf{j} \times \mathbf{B}). \tag{10}$$

Combining (8), (9), and (10), we arrive at a linear "influence parameter" $\hat{h}_i$ which expresses the coupling between the variable that we control, Φ, and the resulting force on the fluid, f_i, in transform coordinates:

$$\hat{f}_i(k_1, x_2, k_3, t) = \hat{h}_i(k_1, x_2, k_3)\, \hat{\Phi}(k_1, k_3, t).$$

In the present problem, control is to be applied to minimize the pressure fluctuation intensity on a section of wall with area A over a period of time T using the least amount of control effort possible. A relevant cost functional the present problem is thus

$$\begin{aligned} \mathcal{J}_p(\Phi) = &\frac{\ell}{2\,A\,T} \int_\Omega \int_{t_0}^{t_0+T} j_i\, j_i\, \frac{1}{\sigma}\, dt\, dV + \\ &\frac{1}{2\,A\,T} \int_w \int_{t_0}^{t_0+T} p'^2\, dt\, dS \end{aligned} \tag{11}$$

where p' is the spatially-fluctuating component of the pressure and w now refers to just the lower wall. The first term on the right hand side is a measure of the power dissipated in the electric currents in the channel—note the integrand is of the form $P = I^2 R$. The second term is a measure of the wall-pressure fluctuation intensity. Again, ℓ is a weighting factor which represents the expense of applying the control—a number which is small if the control is cheap (relative to the importance placed on the pressure fluctuations) and large if it is expensive. Note that this problem is significantly different than the problem studied in Sections 2 and 3: the pressure fluctuations may not be closely correlated to sweep and ejection events. In such a situation in which physical intuition fails to guide us to an effective scheme, the idea of appealing to control schemes which are mathematically based on the control objective is especially attractive.

The <u>flow state</u> U is now governed by (7), the <u>differential state</u> $\dot{U}$ is governed by the Fréchet differential of (7), and the <u>adjoint state</u> $\tilde{U}$ is *defined* by

$$\begin{aligned} -\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial u_j}{\partial x_i} - u_j \frac{\partial \tilde{u}_i}{\partial x_j} - \frac{1}{Re}\frac{\partial^2 \tilde{u}_i}{\partial x_j^2} + \frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_i} &= 0 \\ \frac{\partial \tilde{u}_j}{\partial x_j} &= 0 \end{aligned} \tag{12a}$$

with boundary conditions on the walls

$$\tilde{u}_i = \begin{cases} p' & i = 2, \text{ lower wall} \\ 0 & \text{otherwise} \end{cases} \tag{12b}$$

and initial conditions

$$\tilde{u}_i = 0 \qquad \text{at } t = t_0 + T. \tag{12c}$$

The equation governing $\dot{U}$, which is found by taking the differential of (7a), is linear in the variable $\dot{U}$ and may be written

$$A\dot{U} = \begin{pmatrix} f_i(\tilde{\Phi}) \\ 0 \end{pmatrix}. \tag{13a}$$

The equation governing $\tilde{U}$ in (12a) is linear in $\tilde{U}$ and may be written

$$A^* \tilde{U} = 0. \tag{13b}$$

As in Section 2, the equation used to define the adjoint operator A^* is

$$< A\dot{U}, \tilde{U} > = < \dot{U}, A^* \tilde{U} > + b. \tag{14}$$

Equation (14) may be simplified using (13a), (13b), and the boundary and initial conditions on $\dot{U}$ and $\tilde{U}$ such that the resulting expression is

$$\frac{1}{A\,T}\int_w \int_{t_0}^{t_0+T} p'\,\dot{p'}\,dt\,dS = - \frac{\rho}{A\,T}\int_w \int_{t_0}^{t_0+T} H\,\tilde{\Phi}\,dt\,dS$$

where H may be found by integrating the product of the adjoint velocity field $\tilde{u}_i$ with influence parameter h_i in transform coordinates:

$$\hat{H} = \int_0^{2\delta} \hat{\tilde{u}}_i\,\hat{h}_i\,dx_2.$$

The Fréchet differential of the cost functional $\mathcal{J}_p$ in (11) may be rewritten using this expression and the gradient extracted, which results in

$$\frac{\mathscr{D}\mathcal{J}_p(\Phi)}{\mathscr{D}\Phi} = -\frac{\ell\,\sigma}{A\,T}\frac{\partial\phi}{\partial y} - \frac{\rho}{A\,T}H,$$

With this gradient information, we may formulate a control strategy for the update of Φ as before with equation (6). Implementation of this scheme in a direct numerical simulation of turbulent channel flow is currently being investigated.

It is important to note that an inhomogeneous magnetic field is required in such a configuration to accelerate the fluid. Consider, for the moment, a configuration as in Figure 6 but with no spanwise variation of the wall currents. The magnetic field created in the channel in this case is uniform in space. In such a situation, the forcing can be written as the gradient of some scalar function ψ. As seen in (7a), such a force is entirely balanced by a modification of the pressure field, and *will not* accelerate the flow. When designing flow control configurations, it must be kept in mind that any forcing profile f_i may be broken down using Helmholtz' representation into irrotational and solenoidal parts such that

$$\mathbf{f} = \nabla\psi + \nabla\times(\varphi\nabla\chi)$$

where ψ, φ, and χ are scalar functions. Only the solenoidal component will accelerate the fluid.

It is also important to note that back EMF was not included in this analysis. The forces on the current-carrying wires of the electromagnets by the magnetic field induced by the other electric currents in the system are significant. Such back EMF forces are dependent on the geometry of the wires leading to the individual electrodes and must be taken into account very carefully.

4. CONCLUSIONS

Optimal control theory has been successfully applied to the problem of drag reduction by wall transpiration in turbulent channel flow. Significant drag reduction has been obtained in a direct numerical simulation of this method. The optimal control technique is easily extended to the problem of reduction of wall-pressure fluctuations by EMTC forcing.

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