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NEW FRONTIERS FOR CONTROL IN FLUID MECHANICS: A Renaissance approach

Thomas R. Bewley
Department of MAE, UC San Diego
La Jolla, CA 92093
Email: **bewley@ucsd.edu**

ABSTRACT

As traditional fields, such as controls, mathematics, and fluid mechanics, individually grow towards their maturity, many new opportunities for significant advances lie at their intersection. As a prime example, attempts at effective integration of control theory, Navier-Stokes mathematics, and fluid mechanics are still in their infancy. What is sorely needed is a balanced perspective and understanding in which one both considers flow physics (and the Navier-Stokes equation governing this physics) when designing control algorithms, and, conversely, the requirements and limitations of control algorithms when designing both reduced-order fluid models and the fluid-mechanical systems to be controlled themselves. Such a balanced perspective is elusive, however, as both the research establishment in general and universities in particular are accustomed only to the dissemination and teaching of component technologies in isolated fields. This lecture will briefly survey a few recent attempts at bridging some of the gaps between these disciplines.

INTRODUCTION

Though the number of new directions being taken every year in the broad area of "flow control" is growing almost exponentially, there is still much uncharted territory, and many of the areas which have been investigated at this intersection of technologies still leave fundamental unanswered questions. We will attempt to identify a few of these unanswered questions in this paper. Certainly any paper, such as the present, which attempts to identify "new" frontiers for investigation in a dynamic research area is destined to become outdated quite rapidly. This being said, it is useful to step back for a moment and attempt to gain a bit of perspective on recent work in order to get a glimpse of the directions such research might be leading. To this end, this paper will describe mostly the projects with which the author has been directly involved and will attempt to weave the story which threads these projects together as part of the fabric of a substantial new area of interdisciplinary research.

Space does not permit the complete development of these projects in the present paper; rather, the paper will just hint at several of the more significant results. The reader is referred to the appropriate full journal articles for all of the relevant details and careful placement of these projects in context with the works of others; reprints and preprints of all references by the author and discussed herein are available at at **http://turbulence.ucsd.edu/~bewley**.

Space limitations also do not allow this brief paper to adequately review the recent directions all my friends and colleagues are taking in this field. Rather than attempt such a review and fail, the reader is referred to recent review papers which, taken together, themselves span only a fraction of the current work being done in this active area of research. From the experimental perspective, the reader is referred specifically to recent reviews of Ho & Tai (1996, 1998), McMichael (1996), Gad El Hak (1996), and Moin & Bewley (1994). From the mathematical perspective, the reader is referred to the recent dedicated volumes compiled by Banks (1992), Banks, Fabiano, & Ito (1993), Gunzburger (1995), Lagnese, Russell, & White (1995), and Sritharan (1998), for a sampling of recent results.

LINEARIZATION – Life in a small neighborhood

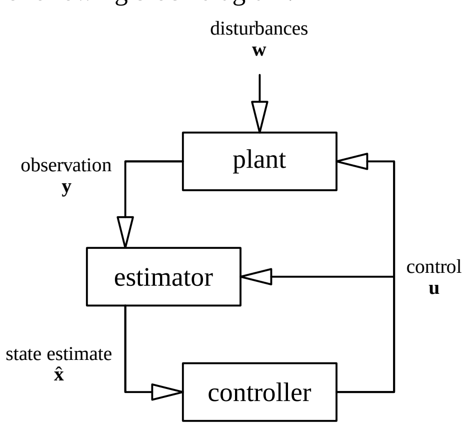
As a starting point for the introduction of control theory into a fluid-mechanical setting, we may consider the linearized system arising from the equation governing small perturbations to a laminar flow. From a physical point of view, such perturbations are very important, as they are the initial stages of the complex process of transition to turbulence, and thus their mitigation is highly desired. Such systems, once appropriately discretized, may be written in the standard form

$$\begin{aligned} \dot{\mathbf{x}} &= A\,\mathbf{x} + B_1\,\mathbf{w} + B_2\,\mathbf{u} \\ \mathbf{y} &= C_2\mathbf{x} + D_{21}\mathbf{w}, \end{aligned}$$

where $\mathbf{x}$ denotes the state, $\mathbf{w}$ denotes the disturbance, $\mathbf{u}$ denotes the control, and $\mathbf{y}$ denotes the available measurements. Not surprisingly, there is a wide body of literature surrounding how to control a linear system of this form. The application of one popular technique based on minimization of certain transfer function norms, called linear optimal/robust control theory, is discussed in detail in Bewley & Liu (1998). The application of another popular technique, called proportional-integral (PI) control, is presented in Joshi, Speyer, & Kim (1997).

Matrices A arising from the discretization of systems in fluid mechanics are often highly "non-normal", which means that the eigenvectors of A are highly nonorthogonal. This is especially true for transition in a plane channel, for which very important characteristics of the system, such as $O(Re)$ transient energy growth at subcritical Reynolds numbers, simply can not be explained by examination of eigenvalues alone. In such systems, control techniques which account for this eigenvector nonorthogonality, such as those based on transfer-function norms, generally have an advantage over control techniques which account for the eigenvalues only, such as those based on analysis of root-locus plots (Bewley & Liu 1998).

Linear control theory may also be used to introduce estimator-based approaches to the fluid-mechanical setting. The flow of information in such an approach is illustrated schematically in the following block diagram.



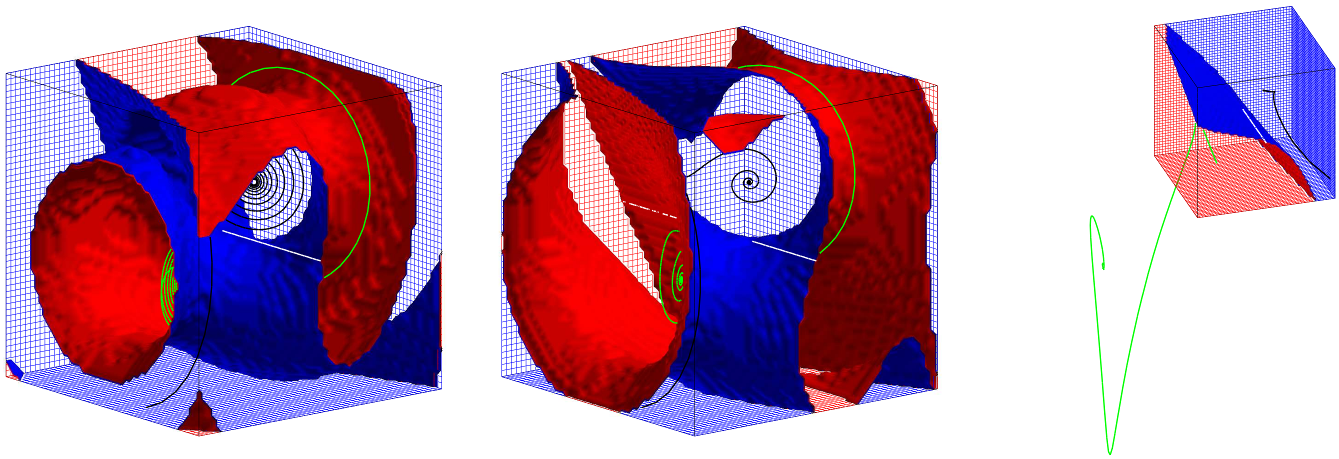
The plant, forced by external disturbances, has an internal state $\mathbf{x}$ which cannot be observed. Instead, a noisy observation $\mathbf{y}$ is made and an estimate of the state $\hat{\mathbf{x}}$ determined. This state estimate is then fed through the controller to determine the control $\mathbf{u}$ to be applied on the plant to regulate $\mathbf{x}$ to zero. Essentially, the full equation for the plant (or a reduced-order model thereof) is used in the estimator as an effective filter to extract useful information about the state from the available (noisy) measurements.

The controllers analyzed in Bewley & Liu (1998) were developed at a particular wavenumber pairs. Recent theoretical work by Bamieh (1997) indicates that taking the inverse transform of an array of such controllers should result in spatial convolution kernels with compact support such that the weights on sensor measurements eventually decay exponentially as a function of distance from the actuators. This property will allow the convolution kernels to be truncated with a prescribed degree of accuracy at a finite distance from each actuator, resulting in implementable schemes in the physical domain. Whether or not this is this case in practice is currently under active investigation.

Another important question which remains open is how best to reduce the equation in the estimator, which must be calculated online in any implementation, to some manageable reduced-order model of the flow. Techniques which retain only a select number of eigenmodes are of dubious value, as the relevance of isolated eigenmodes in such systems is marginal. As mentioned earlier, it is the nonorthogonality of the entire set of eigenvectors which leads to the peculiar (and important) possibilities for energy amplification in these systems. Thus, in addition to the control techniques, model reduction techniques which are mindful of the relevant transfer function norms (such as balanced truncation and Hankel norm approximation) are probably superior, and should be carefully investigated for this problem.

EXTRAPOLATION – Linear control of turbulence???

Even with these follow-on questions remaining open for the moment, we may begin to consider the application of the linear control feedback determined by the linear analysis of Bewley & Liu (1998) directly to the fully nonlinear problem of a turbulent flow. The first reason to try such an approach is simply because we can: due to the ease of determining and implementing linear control feedback, we are veritably bound to exploit everything we can from our ability to compute linear controls. There is at least some evidence in the fluids literature that such an approach may not be entirely crazy. Though the significance of this result has been debated widely in the fluid mechanics community, Farrell & Ioannou (1993) have clearly shown that the linearized Navier-Stokes equation in a plane channel flow, when excited with the appropriate stochastic forcing, exhibits behaviour which is reminiscent of streamwise vortices and streamwise streaks, though it is widely accepted that the creation of such structures in a turbulent flow is an inherently nonlinear process. Whatever information the linearized equation actually contains about the real mechanisms for formation of streamwise vortices and stream-



(a) $\ell = 10$ (b) $\ell = 0.5$ (c) $\ell = 0.025$

Figure 1. Regions of attraction of desired (blue) and undesired (red) stationary points in linearly-controlled convection system and typical trajectories in each region (black and green respectively). The cubical domain illustrated is $\Omega = (-25,25)^3$ in all subfigures; for clarity, slightly different viewpoints are used in each subfigure. As the weighting on the control ℓ is turned down (and the resulting control magnitude increases), the domain of convergence of the undesired stabilized state remains large and this state moves farther from the origin (in a sense, the undesired state becomes aggravated).

wise streaks, the linear controllers should be able to exploit. We proceed on the notion that it's at least worthy of investigation.

In order to understand the possible pitfalls of applying linear feedback to nonlinear systems, a low-order nonlinear convection problem governed by the Lorenz equation was studied in Bewley (1999). As with the problem of turbulent channel flow, but in a very low-order system easily amenable to analysis, we determined the control feedback with linear control theory by linearizing the governing equation about a desired fixed point. Once a linear controller was determined by such an approach, it was then applied directly to the fully nonlinear system. The result is depicted in figure 1.

For control feedback determined from linear control theory with a large penalty on the control in the controller formulation (and thus a small amount of control applied as a result), direct application of linear feedback to the full nonlinear system stabilizes both the desired state (indicated by the black trajectories of figure 1) and an undesired state (indicated by the green trajectories of figure 1). An unstable manifold exists between these two stabilized points, as indicated by the contorted blue/red surfaces in figure 1. Any initial state on the blue side of this manifold will converge to the desired state, and any initial state on the red side of this manifold will converge to the undesired state.

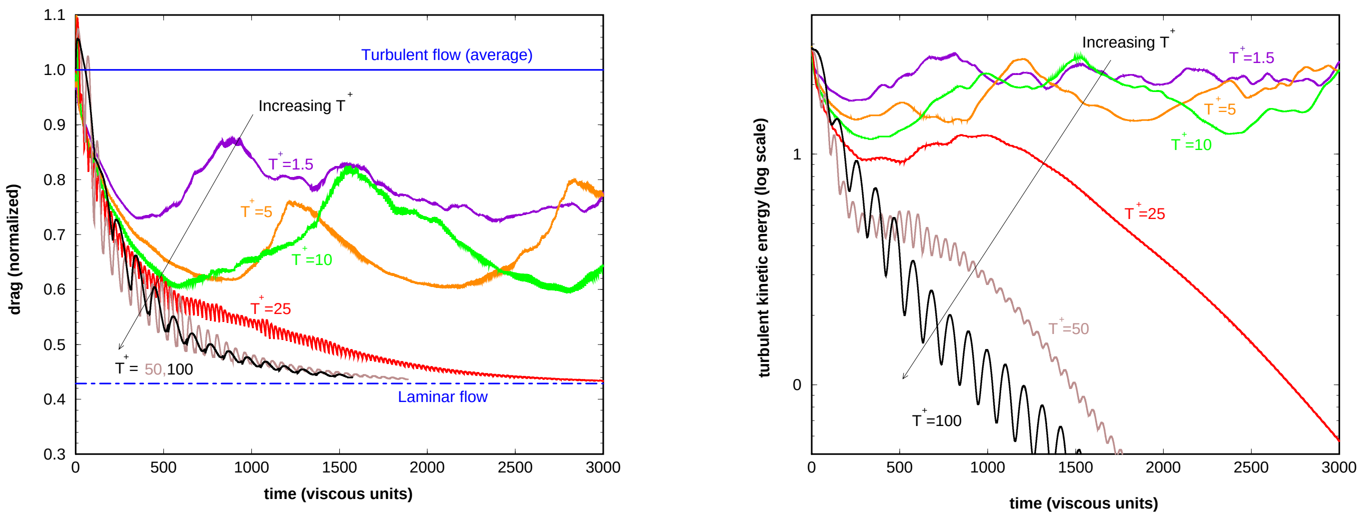
As seen in figure 1, for increased feedback magnitude K (e.g., decreased ℓ), the undesired stabilized state moves farther from the origin, and the domain of convergence of the undesired state remains large; the closed-loop system eventually becomes unbounded for sufficiently large feedback K. Some form of nonlinearity in the feedback rule is required to eliminate this undesired behavior. One effective technique is to apply the control

$$\mathbf{u} = H(R - |\mathbf{x} - \overline{\mathbf{x}}|)\,K(\mathbf{x} - \overline{\mathbf{x}}) \qquad H(\zeta) = \begin{cases} 0 & \text{for } \zeta \le 0 \\ 1 & \text{for } \zeta > 0, \end{cases}$$

such that the control is turned on only when the state $\mathbf{x}(t)$ is inside a sphere of radius R, centered at the desired state $\overline{\mathbf{x}}$, completely contained in the domain of convergence of the desired stationary point in the linearly-controlled system. The chaotic dynamics of the uncontrolled Lorenz system will bring the system into this subdomain in finite time, after which control may be applied to "catch" the state at the desired equilibrium point.

We thus see that even in this very simple model problem, linear feedback can have a destabilizing influence if applied outside the neighborhood for which it was designed. For the full Navier-Stokes problem, though a certain set of linear feedback gains might stabilize the laminar state, on the "other side of the manifold" might lie a turbulent state which is aggravated be the same linear controls. The easy fix found for the convection problem (that is, simply turn off the control until the chaotic dynamics bring the state into a neighborhood of the desired state) is probably not available for the (high-dimensional) problem of turbulence, as fully turbulent flows remain at all times far from the laminar state.

We are thus wary of our current attempts to apply linear control theory to the fully nonlinear problem of turbulence. It has to at least be tried, but there are clear indications that significant difficulties may be encountered. In order to provide guidance as to how feedback control might eventually be applied to mitigate turbulence, we now turn to a less practical but more reliable control framework, referred to in the controls literature as receding-horizon model predictive control.



(a) History of drag.

(b) History of turbulent kinetic energy.

Figure 2. Performance of optimized blowing/suction controls for formulations based on $\mathcal{J}_{\text{TKE(ter)}}$ as a function of the optimization horizon T^+ as computed in direct numerical simulations of turbulent channel flow at $Re_\tau = 100$. For small optimization horizons ($T^+ = O(1)$, sometimes called the "suboptimal approximation"), approximately 20% drag reduction is obtained, a result which can be obtained with a variety of other approaches. For sufficiently large optimization horizons ($T^+ \gtrsim 25$), the flow is returned to the region of stability of the laminar flow, and the flow relaminarizes with no further control effort required. No other control algorithm tested in this flow to date has achieved this result with this type of flow actuation.

OPTIMIZATION – The best case scenario

Given a pristine setting of full flowfield information, no disturbances, and unlimited computer resources, a finite-horizon optimization problem may be formulated and solved for full (nonlinear) Navier-Stokes systems to (locally) minimize a given cost functional which defines the physical problem of interest (Bewley, Moin, & Temam 1999). The general idea of this approach, referred to as receding-horizon predictive control, is best understood by comparing and contrasting it to massively-parallel brute-force algorithms recently developed to play the game of chess. The goal when playing chess is to capture the other player's king through an alternating series of discrete moves with the opponent: at any particular turn, a player has to select one move out of at most fifty or so legal alternatives.

To accomplish its optimization, a computer program designed to play the comparatively "simple" game of chess, such as *Deep Blue* (Newborn 1997), must, in the worst case, plan ahead by iteratively examining a tree of possible evolutions of the game several moves into the future (Atkinson 1993). At each step, the program selects that move which leads to the best expected outcome, given that the opponent is doing the same, in the spirit of a noncooperative game. The version of *Deep Blue* that defeated Garry Kasparov in 1997 was able to calculate up to 200 billion moves in the three minutes it was allowed to conduct each turn. Even with this extreme number of function evaluations at its disposal on this relatively simple problem, the algorithm was only about an even match with Kasparov's human intuition.

An improved algorithm to the brute-force approaches based on function evaluations alone, suitable for optimizing the present problem in a reasonable amount of time, is available because i) we know the equation governing the evolution of the present system, and ii) we can state the problem of interest as a functional to be minimized. Taking these two facts together, we may devise and solve an iterative procedure based on gradient information, derived from an *adjoint field*, to optimize the controls for the desired purpose on the prediction horizon of interest in an efficient manner. Only by exploiting such gradient information can the high-dimensional optimization problem at hand (up to $O(10^7)$ control variables per optimization horizon at $Re_\tau = 180$) be made tractable. It is desirable that the optimization problems we formulate be as well-conditioned as possible in order to make them amenable to efficient gradient-based numerical optimization algorithms. To this end, several different formulations of the present control problem have been considered. Three representative cases are

$$
\begin{aligned}
\mathcal{J}_{\text{DRAG}}(\phi) &= -d_1 \int_0^T \int_{\Gamma_2^\pm} \nu \frac{\partial u_1(\phi)}{\partial n}\, d\mathbf{x}\, dt + \frac{\ell^2}{2} \int_0^T \int_{\Gamma_2^\pm} \phi^2\, d\mathbf{x}\, dt, \\
\mathcal{J}_{\text{TKE(reg)}}(\phi) &= \frac{d_2}{2} \int_0^T \int_\Omega |\mathbf{u}(\phi)|^2\, d\mathbf{x}\, dt + \frac{\ell^2}{2} \int_0^T \int_{\Gamma_2^\pm} \phi^2\, d\mathbf{x}\, dt, \\
\mathcal{J}_{\text{TKE(ter)}}(\phi) &= \frac{d_3}{2} \int_\Omega |\mathbf{u}(\phi;T)|^2\, d\mathbf{x} + \frac{\ell^2}{2} \int_0^T \int_{\Gamma_2^\pm} \phi^2\, d\mathbf{x}\, dt.
\end{aligned}
$$

Clearly, in both the chess problem and the turbulence problem, the further into the future one can optimize the problem the better (figure 2); however, both problems get exponentially harder to optimize as the prediction horizon is increased. Since only intermediate-term optimization is tractable, it is not always the best approach to represent the final objective in the cost functional. In the chess problem, though the final aim is to capture the other player's king, it is most effective to adopt a mid-game strategy of establishing good board position and achieving material advantage. Similarly, if the turbulence control objective is reducing drag, it was found in Bewley, Moin, & Temam (1999) that it is most effective along the way to minimize a finite-horizon cost functional related to the turbulent kinetic energy of the flow, as the turbulent transport of momentum is responsible for inducing a substantial portion of the drag in a turbulent flow. In a sense, turbulence is the "cause" and high drag is the "effect", and it is most effective to target the "cause" in the cost functional when optimizations on only intermediate prediction horizons are possible.

In addition, a smart optimization algorithm allows for excursions in the short term if it leads to a long-term advantage. For example, in chess, a good player is willing to sacrifice a lesser piece if, by so doing, a commanding board position is attained and/or a restoring exchange is forced a few moves later. Similarly, by allowing a turbulence control scheme to increase (temporarily) the turbulent kinetic energy of a flow, a transient may ensue which, eventually, effectively diminishes the strength of the near-wall coherent structures. It was found in Bewley, Moin, & Temam (1999) that terminal control strategies, aimed at minimizing the turbulence only at the end of each optimization period, appear to have an advantage over regulation strategies, which penalize excursions of the turbulent kinetic energy over the entire prediction horizon.

ROBUSTIFICATION – Murphy's Law

Though optimal control approaches possess an attractive mathematical elegance and are now proven to provide excellent results in terms of drag and TKE reduction in fully-developed turbulent flows, they certainly suffer from their share of problems, not the least of which is sensitivity to design point, external disturbances, and modeling errors. In order to abate such system sensitivity, one may appeal to the concepts of robust control theory (Bewley, Temam, & Ziane 1999). Such a theory, one might say, amounts to Murphy's law taken seriously:

> If a worst-case disturbance can disrupt a controlled closed-loop system, it will.

When designing a robust controller, therefore, one should *plan* on a finite component of the worst-case disturbance aggravating the system, and design a controller which is suited to handle even this extreme situation. A controller which is designed to work even in the presence of a finite component of the worst-case disturbance will also be robust to a wide class of other possible disturbances which, by definition, are not as detrimental to the control objective as the worst-case disturbance. Thus, the problem of finding a robust control is intimately coupled with the problem of finding the worst-case disturbance, in the spirit of a non-cooperative game.

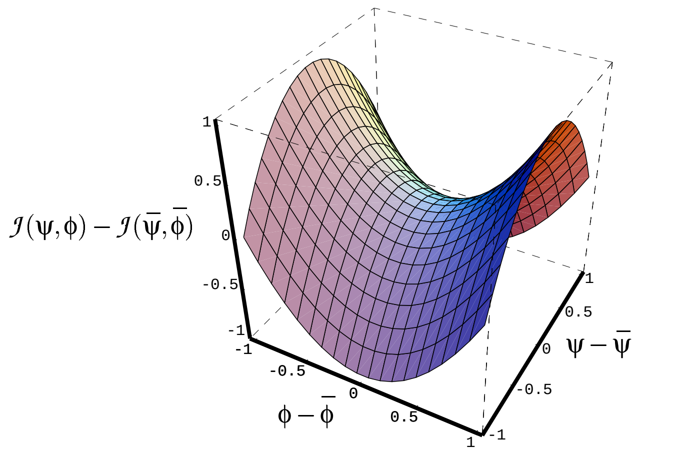


Figure 3. Schematic of a saddle point representing the neighborhood of a solution to a robust control problem with one scalar disturbance variable ψ and one scalar control variable ϕ. When the robust control problem is solved, the cost function $\mathcal{J}$ is simultaneously maximized with respect to ψ and minimized with respect to ϕ, and a saddle point such as $(\bar\psi,\bar\phi)$ is reached. An essentially infinite-dimensional extension of this concept may be formulated to achieve robustness to disturbances and insensitivity to design point in fluid-mechanical systems. In such approaches, the cost $\mathcal{J}$ is related to a distributed disturbance ψ and a distributed control ϕ through the solution of the Navier-Stokes equation.

To summarize briefly the robust control approach in the time domain, a cost functional $\mathcal{J}$ describing the control problem at hand is defined that weighs together the (distributed) disturbance ψ, the (distributed) control ϕ, and the flow perturbation $u(\psi,\phi)$ in the domain Ω over the time period of consideration $[0,T]$. The cost functional considered in Bewley, Temam, & Ziane (1999) is of the form

$$\mathcal{J}(\psi,\phi) = \frac{1}{2}\int_0^T\int_\Omega |\mathcal{C}_1 u|^2\,dx\,dt + \frac{1}{2}\int_\Omega |\mathcal{C}_2 u(x,T)|^2\,dx$$
$$-\int_0^T\int_{\partial\Omega} \mathcal{C}_3 \nu \frac{\partial u}{\partial n}\cdot\vec{r}\,d\Gamma\,dt + \frac{1}{2}\int_0^T\int_\Omega \left[\ell^2|\phi|^2 - \gamma^2|\psi|^2\right]dx\,dt.$$

This cost functional is simultaneously maximized with respect to the disturbance ψ and minimized with respect to the control ϕ, as illustrated in figure 3. The robust control problem is considered to be solved when a saddle point $(\bar\psi,\bar\phi)$ is reached; note that such a solution, if it exists, is not necessarily unique.

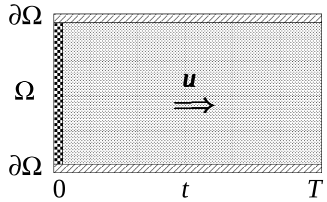


Figure 4. Schematic of the space-time domain over which the flow field u is defined. The possible regions of forcing in the system defining u are:
(1) the right-hand side of the p.d.e., indicated with shading, representing flow control by interior volume forcing (e.g., externally-applied electromagnetic forcing by wall-mounted magnets and electrodes);
(2) the boundary conditions, indicated with diagonal stripes, representing flow control by boundary forcing (e.g., wall transpiration);
(3) the initial conditions, indicated with checkerboard, representing optimization of the initial state in a data assimilation framework (e.g., the weather forecasting problem).

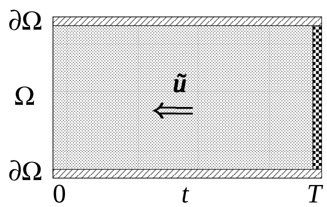


Figure 5. Schematic of the space-time domain over which the adjoint field $\tilde{u}$ is defined. The possible regions of forcing in the system defining $\tilde{u}$, corresponding exactly to the possible domains in which the cost functional $\mathcal{J}$ can depend on u, are:
(1) the right-hand side of the p.d.e., indicated with shading, representing regulation of an interior quantity (e.g., turbulent kinetic energy);
(2) the boundary conditions, indicated with diagonal stripes, representing regulation of a boundary quantity (e.g., wall skin-friction);
(3) the terminal conditions, indicated with checkerboard, representing terminal control of an interior quantity (e.g., turbulent kinetic energy).

GENERALIZATION – A unified framework

The dependence of the cost functional $\mathcal{J}$ on the flow perturbation $u = u(\psi, \phi)$ itself is treated in a fairly general form in the analysis of Bewley, Temam, & Ziane (1999). The three cases discussed previously, related to the regulation of drag and turbulent kinetic energy and the terminal control of turbulent kinetic energy, all may be considered in the present framework, and the extension to other cost functionals is straightforward. Similarly, three different locations of forcing may be identified for the flow problem. As illustrated in figures 4 and 5, the various regions of forcing of the flow and adjoint problems together form a general framework which can be applied to a wide variety of problems in fluid mechanics including both flow control (e.g., drag reduction, mixing enhancement, and noise control) and flow forecasting (e.g., weather prediction and storm forecasting). Related techniques, but applied to the time-averaged Navier-Stokes equation, have also been used extensively to optimize the shapes of airfoils (see, for example, Reuther *et al.* 1996).

By identifying a range of problems which all fit into the same general framework, we can better understand how to extend, for example, the ideas of robust control to the full suite of related problems. In fact, the adjoint field at the heart of all of these approaches may be used for both the minimization w.r.t. the control *and* the maximization w.r.t. the disturbance, so the additional computational complexity added by the robust component of the controller formulation is simply a matter of storage of the appropriate disturbance variables. Many disciplines have noted that adjoint-based optimization strategies tend to "over-optimize" the system, leaving a high degree of design-point sensitivity. The noncooperative framework of robust control provides a natural means to "detune" the optimal result and can be applied easily to a broad range of related applications.

IMPLEMENTATION – Intelligent compromise

In addition to "robustifying" the result, there are several other practical issues which can only be mentioned briefly here.

The direct numerical simulations (DNS) reported in figure 2 were conducted at very low Reynolds number. In order to begin to examine higher Reynolds number turbulent flows, it is imperative that we learn how to apply the optimization framework to large eddy simulations (LES). There are very subtle questions of how to handle the sub-grid-scale (SGS) model in the computation of the adjoint field. The reader is referred to the pioneering work of Chang & Collis (1999) for a discussion of these issues.

Model predictive control, of course, might never actually even be applied in a real turbulent flow. In such a flow, the state evolves extremely quickly—Mother Nature does not stop to allow us the three minutes per update that was allowed to *Deep Blue* in the match with Kasparov. However, there are a variety of implementations in which the framework of model predictive control is still quite useful, including the supercomputer-based optimization of sets of feedback coefficients (to be selected and implemented in a real-world implementations) and the optimization of open-loop time-periodic forcing schedules to establish a desired (optimized) approximately time-periodic response in a given turbulent flow. Such practical applications will be studied in future work once all of the salient details have been sorted out on idealized model problems.

Finally, it is important to mention that it is paramount that reduced-order models of turbulence be developed which are effective in the closed-loop setting. Preliminary work in this direction using the proper orthogonal decomposition (POD) is reviewed by Lumley & Blossey (1998). Note that reduced-order models which are effective in the closed-loop setting need not capture the majority of the energetics of the unsteady flow; future control-oriented models may benefit by deviating from the standard mindset of attempting to capture such energetics.

DISCUSSION – A common language for dialog

It is imperative that an accessible language be developed which provides a common ground upon which people from the fields of fluid mechanics, mathematics, and controls can meet, communicate, and develop new theories and techniques for flow control. Pierre-Simon de Laplace once said

> Such is the advantage of a well constructed language that its simplified notation often becomes the source of profound theories.

Similarly, it was recognized by Gottfried Wilhelm Leibniz that

> It is worth noting that the notation facilitates discovery. This, in a most wonderful way, reduces the mind's labor.

Profound new theories are still possible in this young field. To a large extent, however, we have not yet homed in on a common language in which such profound theories can be framed. Such a language needs to be actively pursued; time spent on identifying, implementing, and explaining a clear "compromise" language which is approachable by those from the related "traditional" disciplines is time well spent. An example problem which might be addressed by such a collaborative dialog is shown in figure 6.

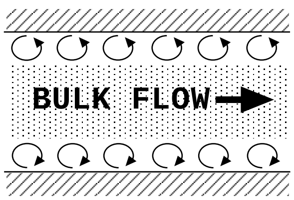


Figure 6. An enticing picture: fundamental restructuring of the near-wall unsteadiness to insulate the wall from the viscous effects of the bulk flow. It has been argued (Nosenchuck 1994, Koumoutsakos 1999) that it might be possible to maintain a series of so-called "fluid rollers" to effectively reduce the drag of a flow, much as a series of solid cylinders can serve as an effective conveyer belt. One problem with this picture is that between each clockwise-spinning roller must exist a roller spinning counter-clockwise, as the fluid is a continuum. It is still the topic of some debate whether or not such a configuration could be maintained by a control which acts solely on the unsteady component of the flow in such a way as to sustain the mean skin-friction below laminar levels. (Such a control might be applied by either interior electromagnetic controls applied by wall-mounted magnets and electrodes or by boundary controls such as blowing/suction.) Definitive answers to questions like this, posed initially in the fluid mechanics community, might be possible by rigorous analysis of the Navier-Stokes equation governing this flow.

There are, of course, some significant obstacles to the implementation of a common language. For example, fluid mechanicians have historically used **u** to denote flow velocities and **x** to denote spatial coordinates, whereas the controls community overwhelmingly adopts **x** as the state vector and **u** as the control. Writing papers in a manner which is conscious to these different backgrounds (for example, explaining the notation adopted and the necessary controls terms to the fluids folks and the fluids concepts to the controls folks) is certainly extra work. However, such an exercise is necessary to make interdisciplinary work accessible to the large audience of people in related fields.

Besides mathematical notation, there are certain reserved words which mean something very specific in various fields. Though this is not necessarily easy, we should learn these words and respect their meanings, even if such words don't carry a strong meaning in the particular field in which we were educated. For instance, the word "optimal" in the controls literature is related to a specific procedure. Though this procedure can take a variety of forms and be solved with either adjoint-based approaches or Riccati based approaches, it is not synonymous with the word "optimized". Optimization can take a variety of model-based or adaptive approaches that may or may not be related to optimal control. Similarly, the word "robust" in the controls literature has a very specific meaning, well-established supporting theory (in fact, it may be posed as an extension of optimal control to a non-cooperative framework), and concomitant implications in terms of disturbance rejection. The word "robust" is not synonymous with "effective" or "durable" and can not be used as such in flow control research without false implications. In addition, the word "model" in the fluids literature usually has the connotation of a reduced-order model, though this implication is not made in the controls literature. Thus, to avoid confusion, the word "model" should be avoided in flow-control research unless specifically referring to a "reduced-order model". Also, the word "flux" in fluid mechanics has the connotation of convective transport of a quantity with the fluid through a given area, or

> The rate of flow of any fluid across a given area; the amount which crosses an area in a given time; it is thus a vector referred to unit area. Also used with reference to other forms of matter and energy that can be regarded as flowing...

(Oxford English Dictionary, 2nd ed.). Usage of this word for the gradient of a quantity at a solid surface should thus be avoided.

In summary, we can not borrow words from disciplines outside of our own without learning and respecting what these reserved words mean. Imagine someone in the controls world picking up a commercial flow code and running a time-averaged simulation of the flow over the shape he was optimizing with a $k-\varepsilon$ model for the turbulence. If he told the flow control community he was doing a "direct numerical simulation" of this flow, he would literally be correct using the words as they are defined in his own field, where they have no special connotation. However, he would be (perhaps unknowingly) misleading the entire fluids community with his poor choice of words. So also are descriptions misleading when words from the controls community are misused by those coming from a fluids background. It is our duty to use caution to avoid such miscommunications before they happen by using terminology consistent with that used by both the fluids community and the controls community when doing flow control research.

Similarly, we should avoid the temptation to invent new names for techniques which are already well established in other disciplines. For example, some of the work I have been involved with uses a technique which has been discussed extensively in the controls literature under the name of predictive control. Were we to invent a new name for such a procedure, being the first to actually apply it in a fluid-mechanical setting, we would perhaps gain more recognition. However, not only would we be misleading our peers with the impression that we were inventing something new, we would not be giving others who came before us proper credit for implementing such a procedure in other disciplines. It is an excellent idea to introduce tools from one discipline into another discipline; such a procedure forms the very foundation for much interdisciplinary work. However, it is not acceptable to claim this tool as something new to those who know no better for the purpose of undue recognition. Instead, it is much more constructive to determine and explain precisely where the technique one has picked up fits in to the existing body of literature.

THE FUTURE – A Renaissance

In order to promote interdisciplinary work, describing oneself as working at the intersection of disciplines X and Y (or, where they are still disjoint, the bridge between such disciplines) needs to become more commonplace. People often resort to the philosophy "I do X... oh, and I also sometimes dabble a bit with Y", as the philosophy "I do $X{*}Y$", where $*$ denotes something of the nature of an integral convolution, has not been in favour since the Renaissance. Perhaps the sole reason for this is that X and Y (and Z, W, ...) have gotten progressively more and more difficult. By specialization (though often to the point of isolation), we are able to "master" our more and more narrowly-defined disciplines. In the experience of the author, not only is it often the case that X and Y are not immiscible, but the solution sought may often not be formulated with the ingredients of X or Y alone. In order to advance, new research must be conducted at the intersection of X and Y; we must prepare ourselves and our students to attack new problems with a Renaissance approach.

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