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ON THE SEARCH FOR FUNDAMENTAL PERFORMANCE LIMITATIONS IN FLUID-MECHANICAL SYSTEMS

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ABSTRACT

A simple pressure-based feedback control strategy for wall-transpiration control of incompressible unsteady 2D channel flow was recently investigated by Aamo, Krstic, & Bewley (2001). Nonlinear 2D channel flow simulations which implemented this control strategy resulted in flow transients with instantaneous drag far lower than that of the corresponding laminar flow. The present note examines the physical mechanism by which this very low level of instantaneous drag was attained. It then explores the possibility of achieving *sustained* drag reductions to below the laminar level by initiating such low-drag transients on a periodic basis. All attempts at sustaining the mean flow drag below the laminar level fail, providing further evidence in favor of the conjecture that the laminar state provides a fundamental "performance limitation" in such flows.

1 THE CONJECTURE

Motivated by the active debate surrounding fundamental performance limitations and certain proposed mechanisms for channel-flow drag reduction, the following, as yet unproven, conjecture was proposed by Bewley[2]:

Conjecture: *The lowest sustainable drag of an incompressible constant mass-flux channel flow in either 2D or 3D, when controlled via a distribution of zero-net mass-flux blowing/suction over the channel walls, is exactly that of the laminar flow.*

Note that, by the "sustainable drag" (denoted D_∞), we mean the time average (denoted $\overline{D}(T)$) of the instantaneous drag (denoted $D(t)$) as the averaging time T approaches infinity, *i.e.*,

$$D_\infty \triangleq \lim_{T\to\infty} \overline{D}(T) \triangleq \lim_{T\to\infty} \int_0^T \frac{D(t)}{T}\,dt \triangleq \lim_{T\to\infty} \frac{-\mu}{T} \int_0^T \int_{\Gamma_2^\pm} \frac{\partial u}{\partial n}\,d\mathbf{x}\,dt,$$

where $\mathbf{n}$ is an outward facing normal, $\Gamma_2^\pm$ denotes the set given by the union of the upper and lower walls of the channel, μ is the viscocity, u is the streamwise component of the velocity vector $\mathbf{u}$, and D_L denotes the drag of the corresponding laminar channel flow with the same dimensions, viscosity and bulk velocity.

Recent 2D simulations of controlled channel flows demonstrating strong $D(t) < D_L$ transients[3;1] have cast some doubt as to the validity of this conjecture. The purpose of this note is to investigate the mechanism behind these transients and the possible utilization of this mechanism to provide sustained drag reductions to sublaminar levels.

2 THE TRANSIENT LOW-DRAG MECHANISM

The following feedback control rule was proposed and tested in Aamo, Krstic, & Bewley[1]:

$$\phi^\pm = k\,(p^\pm - p^\mp), \tag{1}$$

where $\phi^\pm = -\mathbf{u}\cdot\mathbf{n}$ is the blowing/suction distribution which is applied to the walls $\Gamma_2^\pm$ of the channel flow system as the control, $p^\pm$ is the pressure on the corresponding wall, $p^\mp$ denotes the pressure on the opposite wall, and k is a constant. With such a strategy, blowing at one wall of the channel is always countered with suction of equal magnitude at the opposite wall. A feedback rule of this form was motivated by analysis of the energetics of the channel flow system at extremely low Reynolds number; see Aamo, Krstic, & Bewley[1] for details.

Regardless of the motivation for considering the feedback rule (1), it is of interest here to study the flow that results when (1) is applied to the 2D channel flow system at supercritical Reynolds numbers. It was observed in Aamo, Krstic, & Bewley[1] that feedback of this type, when applied to the fully established unsteady flow in a 2D channel at $Re = 7500$, resulted in a flow transient with drag far below the laminar level. A similar transient was also observed in earlier work by Cortelezzi, Lee, Kim, & Speyer[3], where a low-drag transient to 50% below the laminar level was reported in a 2D flow.

As reported in Figures 1-4, a transient which actually achieves *negative* total drag for a short period of time is achieved by applying (1) to a fully-established, unsteady, constant mass-flux 2D channel flow at Re=7500. Jiménez[4] describes the uncontrolled 2D flow system. The simulation reported here used a box length of 60 times the channel half width at a resolution of 1024×128 using the DNS code of Lumley & Blossey[5].

The flow at $t = 0^-$ in Figure 1, a fully established unsteady flow in a 2D channel, has extensive regions of backflow near the walls. This appears to be the key to initiating a $D(t) < D_L$ transient. A scatter plot of the local control ϕ as a function of the local value of drag $(-\mu\partial u/\partial n)$ at $t = 5$ (shortly after the control is turned on) is shown in Figure 2, demonstrating correlation of blowing with local regions of positive drag and suction with local regions of negative drag using the present strategy (76% of the samples are in the first and third quadrants). By generally applying suction at the walls in regions of negative drag, and applying blowing in regions of large positive drag, the negative drag regions are intensified (locally, more negative drag) and the high positive drag regions are moderated (locally, less positive drag), as illustrated in Figure 3. In terms of reducing the total instantaneous drag $D(t)$ integrated over the walls at time $t = 5$, both effects are beneficial, and thus the control application results, for a brief amount of time, in a "win-win" situation, facilitating a drastic transient reduction in skin-friction drag to well below laminar levels. Unfortunately, the wall suction quickly acts to remove the backflow from the flow domain entirely, after which the instantaneous drag $D(t)$ asymptotes back to the laminar level D_L.

A metric which quantifies the backflow present at any instant in a particular flow is given by $b_p = \left[\frac{1}{V}\int_{\Omega^-} |u|^p d\mathbf{x}\right]^{1/p}$, where Ω^- is the subset of the channel flow domain Ω which is characterized by regions of flow with negative streamwise velocity, *i.e.*, $\Omega^- = \{\Omega(x,y) | u(x,y) < 0\}$, and V is the volume of the entire channel domain Ω. For the simulation depicted in Figures 1-3, plots of the history of b_1 and b_2 are shown in Figure 4. Note that, by both measures, the backflow is quickly eliminated after the control is initiated; flow visualizations such as Figure 3 demonstrate that the backflowing fluid in Ω^- is simply removed from the channel by the control suction.

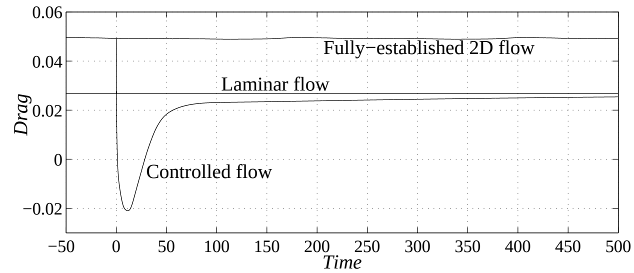

FIGURE 1. History of drag. Simulation initated from fully established unsteady 2D flow[4] at $Re = 7500$. Stabilizing pressure-based feedback control strategy (1) with $k = 0.125$ turned on at $t = 0$.

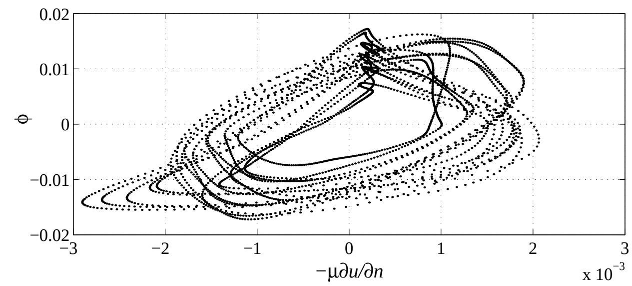

FIGURE 2. Scatterplot of ϕ versus $(-\mu\partial u/\partial n)$ at $t = 5$.

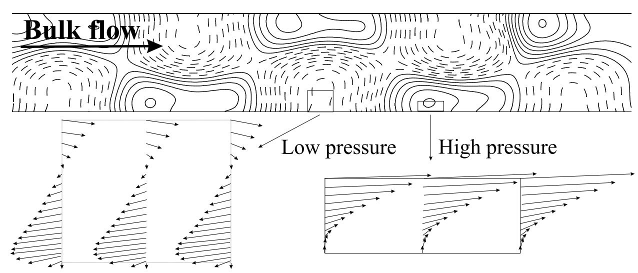

FIGURE 3. Win-win mechanism at $t = 5$: intensification of local regions of negative drag by suction in low pressure regions and moderation of positive drag by blowing in high pressure regions. Shown are contours of pressure in 1/6 of the computational domain (top) and selected velocity profiles (bottom).

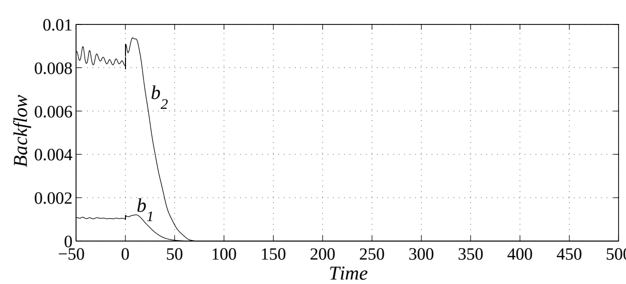

FIGURE 4. Elimination of backflow after control is turned on, as measured by $b_1(t)$ and $b_2(t)$.

Case	T_{cycle}	T_1	T_2	k_1	k_2
1	3000	2600	400	0	0.125
2	3000	2700	300	0	0.125
3	3000	2800	200	0	0.125
4	3000	2900	100	0	0.125
5	3000	2950	50	0	0.125
6	3000	2000	1000	0	0.031
7	3000	2500	500	0	0.031
8	3000	2800	200	0	0.031
9	2000	1600	400	0	0.125
10	2000	1700	300	0	0.125
11	2000	1800	200	0	0.125
12	2000	1900	100	0	0.125
13	2000	1950	50	0	0.125
14	1000	325	675	-0.031	0.031
15	1000	350	650	-0.031	0.031
16	1000	500	500	-0.031	0.031

TABLE 1. Forcing schedules explored during parametric study: T_{cycle} indicates the period of the cycle used (in units of δ/U_c), T_1 denotes the duration of the first segment of the cycle, T_2 denotes the duration of the second segment, k_1 denotes the feedback coefficient used during the first segment, and k_2 denotes the feedback coefficient during the second segment. All simulations were initialized from a slightly perturbed laminar flow. Note that δ is the channel half width and U_c is the centerline velocity of the corresponding laminar flow.

3 CYCLING THE CONTROLLER OFF AND ON

As a "standard" problem to test the utility of a given control strategy for reducing time-averaged drag to below laminar levels, a series of controlled 2D channel flow simulations at $Re = 7500$ were initialized from small (random) perturbations to a laminar flow profile. The control producing the $D(t) < D_L$ transients was cycled off and on periodically, with the "running average" of the drag, $\overline{D}(t) = \int_0^t D(t')\,dt'$, computed as the flow evolved to quantify progress towards sustained drag reduction. A large variety of different periods, duty cycles, and control amplitudes were explored; Table 1 summarizes specific cases examined in detail.

Cases 1-5 reported in Table 1 were executed at a cycle time of $T_{cycle} = 3000$ for a variety of duty cycles with relatively strong stabilizing feedback applied during the second segment of each cycle. Cases 6-8 were similar, but applied relatively weak stabilizing feedback. Cases 9-13 returned to the relatively strong stabilizing feedback, but investigated a shorter cycle time. Finally, cases 14-16 were executed with destabilizing feedback applied during the first segment of each cycle, and stabilizing feedback applied during the second segment of each cycle; this was done to accelerate the formation of the backflow regions. Histories of the L^2 energy, the instantaneous and "running time-averaged" drag $D(t)$ and $\overline{D}(t)$, and the backflow measures b_1 and b_2 are illustrated in Figure 5 for four representative cases.

It was found in cases 1, 2, 9, 10, and 14, with T_2 relatively *large*, that the stabilization provided by the control during the second segment of each cycle was sufficient to stabilize the entire channel flow back to the parabolic profile; to illustrate, case 14 is plotted in Figure 5c. These cases imply that T_2 must be a sufficiently small fraction of T_{cycle} in order to allow a quasi-periodic behavior to establish.

It was found in cases 5, 8, 13, and 16, with T_2 relatively *small*, that the uncontrolled (or, in case 16, destabilized) evolution of the flow during the first segment of each cycle was sufficient to drive the time-averaged drag to heightened levels.

A tradeoff is thus identified: decrease T_2 and there will be more backflow to exploit during each cycle (so the transient will be more effective at reducing drag), but by allowing the 2D unsteady flow to evolve for a longer time uncontrolled or destabilized, the mean drag is pulled up higher above the laminar level. Intermediate values of T_2 were sought for a variety of cycle times and forcing amplitudes over a parametric study of several simulations, some of which are reported here. Over all these simulations, this tradeoff was evident, and *not once did the running average, $\overline{D}(t)$, dip below the laminar value* when the simulations were initiated from the perturbed laminar state. These results indicate that it is always necessary to pay a more expensive price (in terms of the time-averaged drag) to obtain the backflow than the benefit (in terms of the time-averaged drag) that can be obtained by applying suction to the backflow regions.

4 POSITIVE ASPECTS OF PRESENT RESULTS

For the first time, drag reduction to below the laminar level in a channel-flow system with zero-net blowing/suction controls has been:

a) obtained with an extremely *simple* feedback control strategy [see (1)] motivated by global analysis of the nonlinear Navier-Stokes equation[(1)];
b) *understood* and explained with a clear physical mechanism and flow visualizations;
c) *fully resolved* with the proper grid refinement studies (Note the usage of grid resolutions of 1024x128 and box lengths of 60 channel half widths. The results were confirmed with two completely separate and previously benchmarked DNS channel-flow codes.); and
d) *identified* for what it is, a fleeting transient.

In addition, for the first time, instantaneous total drag (integrated over the channel walls) in a constant-mass-flow 2D channel flow has been driven to *negative* levels. This is a remarkable result, but is fully consistent with the properly-stated conjecture summarizing the common intuition of the bulk of the fluid mechanics community.

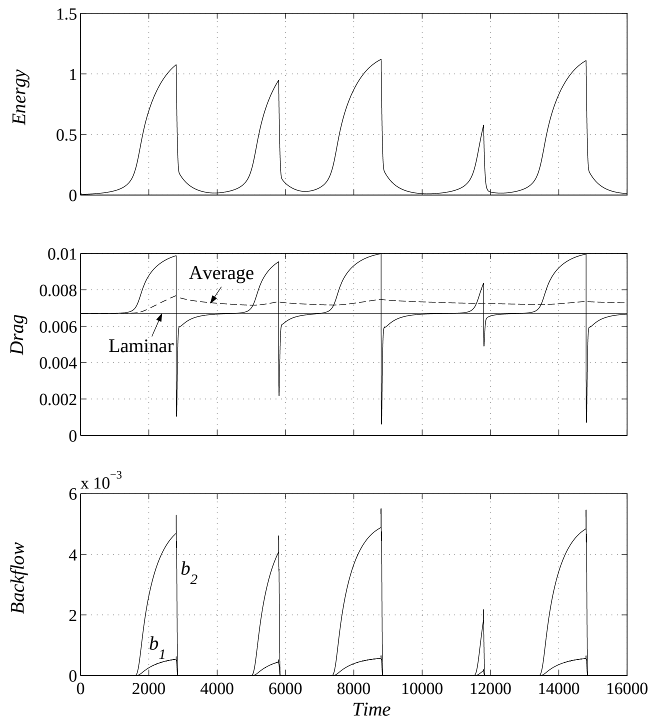

FIGURE 5a. Case 3: $T_{cycle} = 3000$, no feedback for $T_1 = 2600$, relatively strong stabilizing feedback for $T_2 = 400$.

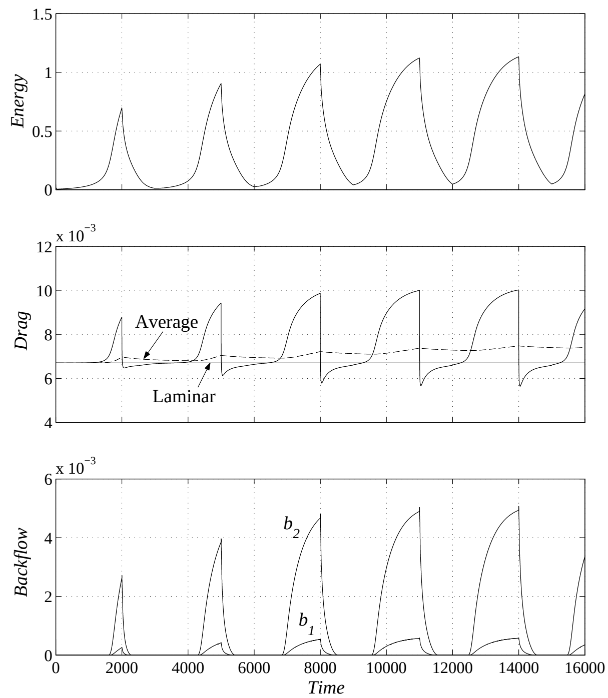

FIGURE 5b. Case 6: $T_{cycle} = 3000$, no feedback for $T_1 = 2000$, relatively weak stabilizing feedback for $T_2 = 1000$.

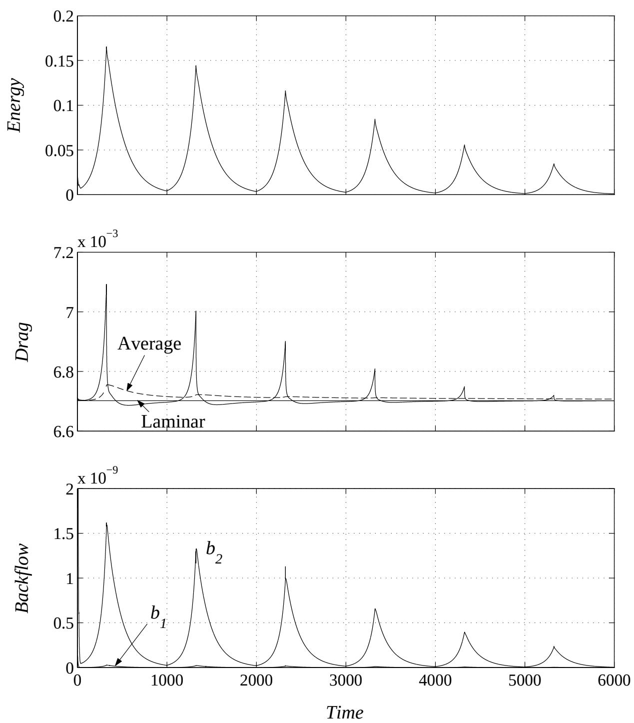

FIGURE 5c. Case 14: $T_{cycle} = 1000$, destabilizing feedback for $T_1 = 325$, stabilizing feedback for $T_2 = 675$.

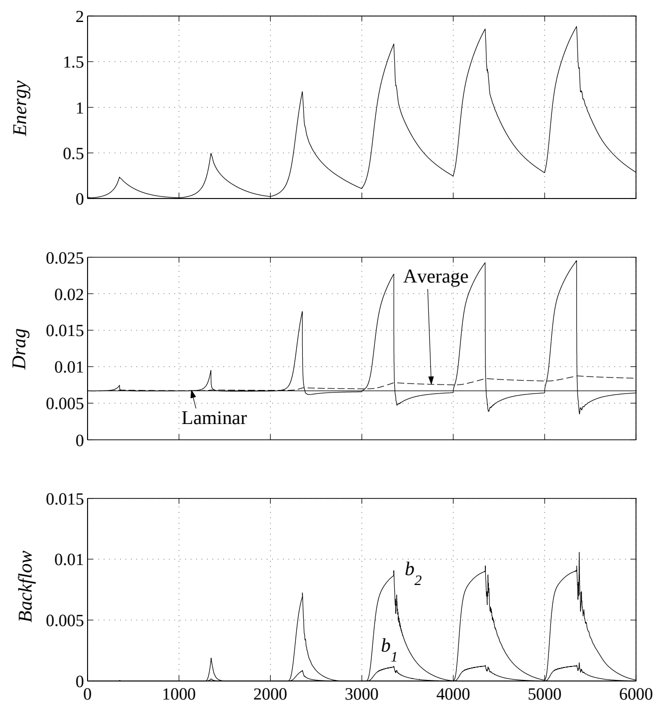

FIGURE 5d. Case 15: $T_{cycle} = 1000$, destabilizing feedback for $T_1 = 350$, stabilizing feedback for $T_2 = 650$.

5 THE UTILITY OF NEGATIVE RESULTS

Though the traditional scientific culture generally discourages it, sometimes it is just as important to report *negative* results as it is to report positive results. A classic example of this is the Michelson-Morley experiment which failed to establish the existence of the ether. The “failure” of the supposed goal of this experiment predated Einstein’s theory of special relativity explaining the results of this experiment by several years. Indeed, the negative results of this experiment provided impetus for Lorentz, Poincaré, and eventually Einstein to question Maxwell’s classical theory of electromagnetism.

Evidence of a similar negative nature is provided in the present note. Though we can now clearly understand a mechanism which provides $D(t) < D_L$ transients in 2D channel flows, a thorough parametric study of simulations which chain such transients together all indicate the inability of this mechanism to sustain time-averaged drag below laminar levels. The present results thus point consistently towards, but do not prove, the conjecture concerning the possible fundamental performance limitation implied by the laminar flow solution, even in light of recent explorations of flow control strategies demonstrating dramatic $D(t) < D_L$ transients.

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