IMPLEMENTATION OF BOOSTCONV TO ACCELERATE THE OPPOSITELY-SHIFTED SUBSPACE ITERATION (OSSI) METHOD FOR APPROXIMATE OPTIMAL CONTROL WITHOUT MODEL REDUCTION

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The Oppositely-Shifted Subspace Iteration (OSSI) method proposed and described in [1] (to which the reader is referred for details) provides a promising new method for approximate optimal control of large linear (or, linearized) systems, such as those derived from flow stability problems, while bypassing the intermediate (and, as it turns out, unnecessary) step of open-loop model reduction, which can oft be problematical. The OSSI method accomplishes this via:

1) iteratively computating Schur vectors corresponding to the least-stable of the stable eigenvalues of the (block 2×2) Hamiltonian matrix Z corresponding to the optimal control problem in question,

2) block decomposing the top and bottom halves of this set of Schur vectors as, respectively, X and P (corresponding to the state and adjoint components of the Schur vectors computed), then

3) approximately solving the corresponding Algebraic Riccati Equation by forming the product $W = PX^+$, where $()^+$ denotes the Moore-Penrose psuedoinverse.

The OSSI family of method achieves step (1) by implementing subspace iterations with "opposite shifts"; that is, by marching the state part of the approximate Schur vectors one direction in pseudotime, while marching the adjoint part of the approximate Schur vectors the opposite direction in pseudotime. This simple idea tends to identify quickly the central Schur vectors of the Hamiltonian matrix Z as the subspace iterations proceed.

In the case that all of the Schur vectors corresponding to the stable eigenvalues are found via this approach, X is square, $X^+ = X^{-1}$, and $W = PX^{-1}$, so the full optimal control problem is solved. In the case that only a subset of the these Schur vectors are found, the Riccati equation in question (and, the corresponding optimal control problem) is only approximately solved by the value of W computed. The choice to include in this computation the Schur vectors corresponding to the least-stable of the stable eigenvalues is based on the hypothesis that, often, these Schur vectors include the components that are the "most important" in the PDE control problems of interest. By performing the "approximating" step during the Riccati solution, rather than beforehand during a separate model reduction step (based, typically, solely on open-loop criteria), the control objective itself is accounted for during the approximation.

The BoostConv method proposed and described in [2] (to which, again, the reader is referred for details) provides an accelerated iterative procedure, inspired by Krylov-subspace methods, to solve certain PDE-based subproblems, such as those involved in computing the Schur vectors described above. BoostConv is based on the minimization of a residual norm at each iteration step with a projection basis updated at each iteration rather than at periodic restarts like in the classical GMRES method. This powerful idea is able to stabilize any dynamical system without increasing the computational time of the original numerical procedure used to solve the governing equations. Moreover, it can be easily inserted into a pre-existing relaxation procedure, such as OSSI, with a call to a single black-box subroutine.

The application of BoostConv to accelerate OSSI (that is, subspace iteration methods with opposite shifts implemented) for the approximate solution of flow stability and control problems will be discussed at the conference.

References

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[2] V. Citroa, P. Luchini, F. Giannetti, F. Auteri Efficient stabilization and acceleration of numerical simulation of fluid flows by residual recombination *Journal of Computational Physics* 344, 234?246. 2017.