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Turbulent channel flow estimation

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1 Introduction

Flow control of fluid mechanical systems has received much interest in recent years due to the possible benefits. By, for example, extending the laminar flow region over a wing huge savings in terms of fuel consumption could be made. Many different strategies to control and reduce the drag in turbulent flows have been proposed and attempted. Over the years the approaches have gone from more intuition based toward more systematic and automated approaches. When computing optimal feedback control based on modern control theory complete flow state is required which is not accessible in real applications. However within the same framework one can formulate the related estimation problem where the flow is reconstructed based on only limited information about the flow, for example, wall measurements. Reviews on recent efforts in flow control with model-based feedback control as well as other strategies can be found in, for example, Bewley [1], Kim [7], and in Högberg *et al.* [5].

This work aims at estimating a turbulent channel flow at $Re_\tau = 100$ based on a time history of noisy wall measurements of the flow. We do this by applying an extended Kalman filter based on the linearized Navier–Stokes equations together with a stochastic model based on statistics gathered from a direct numerical simulation (DNS) of the same turbulent flow we aim to estimate. By using relevant statistical information when constructing the stochastic model we get well resolved estimation gains for all measurements and we get an improved estimation process compared with simpler choices of stochastic models.

The present work is divided in three parts. In the first part statistical data is collected from DNS of turbulent Poiseuille flow. That data is used in the second part of the study where we compute optimal estimation gains. In the third part the gains are tested when estimating a turbulent flow.

2 Theory

To apply linear control and estimation theory to the Navier–Stokes equations, we need to represent the equations with a stochastic dynamic system which can be written on the form

$$\begin{aligned} \dot{u} &= Au + Bf, \quad u(0) = u_0, \\ y &= Cu + g, \end{aligned} \tag{1}$$

where u is the state, A is the linear operator including the dynamics of the system, B is the operator acting on the forcing f to the system, and u_0 is the initial condition. Operator C extracts the measurements from the state and g adds stochastic measurement noise with given statistical properties which leaves the actual measured quantity in y. See for example Lewis and Syrmos [8] for details on linear control theory.

By linearizing the Navier–Stokes equations about a turbulent mean flow profile we can identify operator A as the Orr–Sommerfeld/Squire operator. An important issue when estimating a system like (1) is the modelling of the random forcing f. In previous studies where similar estimation techniques have been applied to fluid mechanical problems, for example, Bewley and Liu [2], Högberg *et al.* [5], and [6] f has been assumed to be uncorrelated in space. The aim with this study however, is to construct the covariance R of the random forcing f, such that it in a statistical sense, represents as much as possible of the physics neglected in the linearized model and subsequently to quantify what impact it will have on the estimation process. In Hœpffner *et al.* [4] f is modelled for transitional flows.

We define the estimator of system (1) as

$$\begin{aligned} \dot{\hat{u}} &= A\hat{u} + F(\hat{u}) - v, \quad \hat{u}(0) = \hat{u}_0, \\ \hat{y} &= C\hat{u}, \end{aligned} \tag{2}$$

where $\hat{u}$ is the estimated state, $F(\hat{u})$ contains the terms left out when linearizing Navier–Stokes equations (extended Kalman filter), and $\hat{y}$ is the measurement of the estimated flow. The volume forcing $v = L(y - \hat{y})$ drives the estimator towards the real flow by multiplying the precomputed optimal gains L with the measurement error between the real and estimated flow. By improving the stochastic model of f the idea is that the new estimation gains will work more efficiently than the ones computed with a simpler stochastic model. In [3] details about the estimation and control problem can be found.

3 Results and discussion

The covariance of the forcing term $f = (f_1, f_2, f_3)^T$ is sampled during a long DNS calculation to make the second-order statistics converge. In Figure 1 the real (left) and imaginary part (right) of the covariance of f for wavenumber $(k_x = 1.5, k_z = 6.0)$ is plotted. The variance of the forcing terms are stronger toward the walls as expected and the covariance quickly decreases as the wall-normal distance between points increases. Covariance data for other wavenumber pairs show the same qualitative behavior.

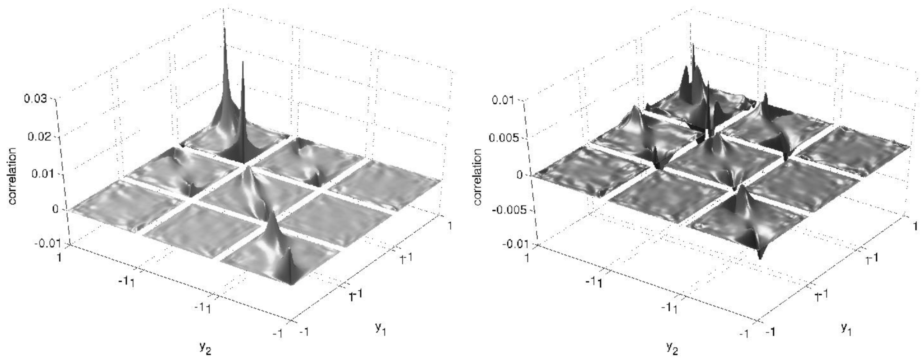

Figure 1: Covariance data for wavenumber pair $k_x = 1.5$, $k_z = 6.0$. The nine squares correspond to the correlation between different components of the forcing vector. From top to bottom the squares are f_1, f_2, and f_3 on each axis. The side of each square represents the channel flow width.

The covariance data is used when we compute the optimal estimation gains L. The simpler stochastic model used in previous studies causes problem when solving the estimation problem and it is not possible to get proper realization of estimation gains for all wall measurements that we can actually measure in an experiment whereas with our physically relevant stochastic model we can retrieve gains for all measurements. Using all gains improves the estimator performance markedly.

To evaluate the performance of the estimator we run two DNS in parallel with roughly uncorrelated initial conditions. One simulation represents the real flow and the other represents the estimated flow where the volume force is applied based on the precomputed estimation gains together with wall measurements in both flows. When the measurements in the two flows converge the volume force becomes weaker. The time averaged correlation between the real and estimated flow is shown in Figure 2 for the cases when using both our improved stochastic model and the older model. The correlation in the figure is defined as

$$\mathrm{corr}(u, \tilde{u}) = \frac{\int_0^{L_x} \int_0^{L_z} u\tilde{u}\, \mathrm{d}x\mathrm{d}z}{\left(\int_0^{L_x} \int_0^{L_z} u^2\, \mathrm{d}x\mathrm{d}z\right)^{1/2} \left(\int_0^{L_x} \int_0^{L_z} \tilde{u}^2\, \mathrm{d}x\mathrm{d}z\right)^{1/2}}. \tag{3}$$

4 Conclusion

By making use of statistical information about the full nonlinear system and including that information into the estimation gain computation we get a better estimator both measured in terms of maximum correlation as well as how far into the channel the correlation reaches.

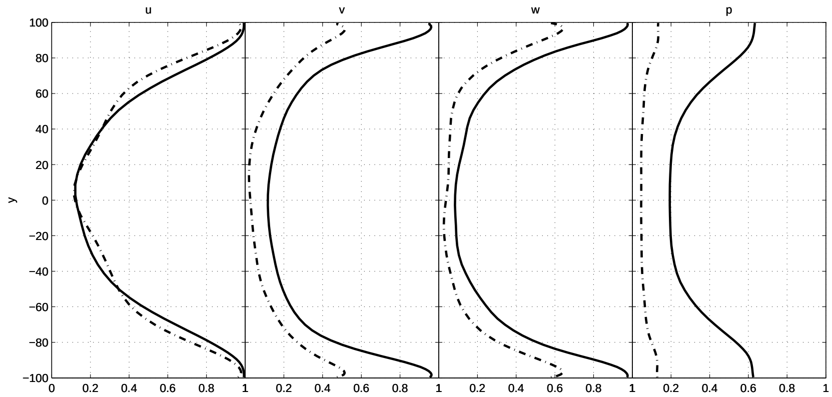

Figure 2: Correlations between real and estimated flow, along the y-axis. The solid line denotes estimation performed with three measurements and gains based on turbulence statistics. The dash-dotted line denotes the estimator performance using only one measurement as considered in Högberg *et al.* [5] where a spatially uncorrelated stochastic model was used.

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