OPTIMAL AND ROBUST CONTROL AND ESTIMATION OF TRANSITION, CONVECTION, AND TURBULENCE

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Preface

"In order to know the true science of the flight of birds,
one must first know the science of the winds."

Man has been at the same time fascinated and stymied by the enigma of fluid turbulence for hundreds of years. In the above quote and several of his drawings (see figure 1), Leonardo da Vinci eloquently expressed man's yearning to understand chaotic fluid motion. Turbulent flow phenomena are ubiquitous and consequential, both in nature and in flows around and through man-made devices. Particular flows of interest in engineering include those of air, water, blood, molten metal, combustion reactants, and hot combustion products. The large increases in drag, cyclic structural loading, internal stresses, mixing, noise, and heat transfer caused by turbulence in such flows have motivated engineers to study turbulence and attempt to alter its effects. However, turbulence is inherently a multi-scale phenomenon, in which fluid motion at the large spatial scales is inherently coupled with fluid motion at the small spatial scales by the complex, chaotic physical process of vortex stretching. This makes turbulence difficult to characterize in simple terms and difficult to predict with low-dimensional simulations. To properly understand, simulate, and, ultimately, control a turbulent flow, this multi-scale nature of turbulence must be accounted for properly.

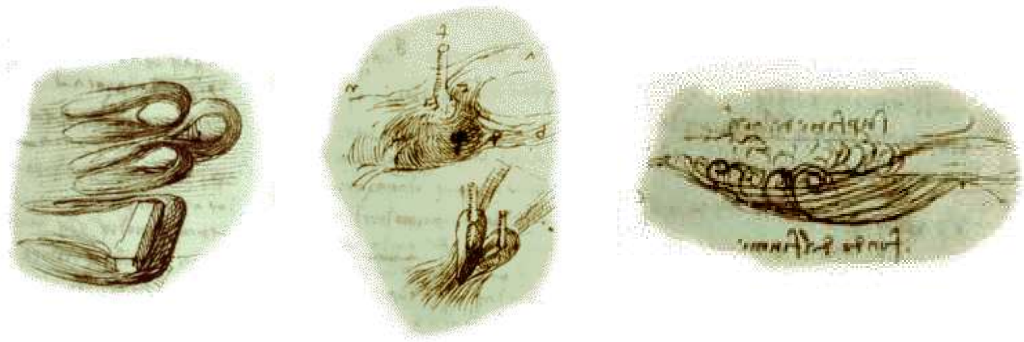

Figure 1: Sketches of turbulent flows from Leonardo da Vinci's *Codex Leicester*, circa 1508.

Turbulent flows are often quite sensitive to both active control and control/optimization by passive techniques, and have a complex, sometimes large, and (in certain respects) predictable response to system disturbances. Unfortunately, our knowledge of how best to determine, suppress, and/or exploit the sensitivities of turbulent flows for particular engineering objectives is, in most cases, quite limited. These sensitivities must be understood and harnessed before current approaches to the design of multi-scale flow systems can be advanced. Such is the topic of distributed control.

The current state-of-the-art in many arenas of control and design optimization for complex flow systems, such as the prevention of combustion instabilities in jet engines, is to conservatively probe the system, trying what seems plausible based almost on the intuition of the engineer alone. For a low-dimensional problem dominated by simple behaviour in the hands of an experienced engineer, such approaches are usually effective. However, as we move to the design of multi-scale, coupled, inherently nonlinear problems, such as the coordination of large arrays of turbulence control devices, intuition can fail us, and a mathematical approach to the design problem is preferred. The purpose of the present work is to establish and demonstrate such an approach for the problem of control of wall-bounded turbulence.

The nonlinear partial differential equation believed to govern turbulent flows, now known as the Navier-Stokes equation, was derived well over a century ago. The mathematical properties of solutions to this seemingly innocuous equation are extremely difficult to characterize, and constitute the primary topic of dozens of books and literally thousands of research papers. Topics of interest include the determination of when chaotic, bounded solutions to the Navier-Stokes equation even exist, when such solutions are unique, and how optimized controls for flows governed by the Navier-Stokes equation might be found. The mathematical approach to answering these questions is very involved and leads to (among other things) the rigorous foundation for the present work; however, this analysis also leads to some insights which are askew with the physical picture. Reconciliation of two viewpoints, drawing both from our *mathematical understanding* of the Navier-Stokes equation and from our *physical understanding* of turbulent flows, is an important part of making turbulence control an engineering reality.

The ability to simulate turbulent flows in simple geometries, integrating the delicate governing equation in time, was mastered well over a decade ago. With the benefit of such simulations, the fundamental physics of certain classes of turbulent flows is now well understood. For example, the driving instability of near-wall turbulence is now known to be closely linked to the chaotic interaction of the vortical regions of near-wall fluid, which are created by the mean shear of the viscous flow near the wall. The regions of the flow with high vorticity near the wall, commonly referred to as the coherent structures, have a characteristic behaviour that has been carefully studied. These vortical regions often appear as “hairpins” and longitudinal “tubes”, as shown in figure 2, that tangle, interact, and evolve in a complex, intermittent fashion.

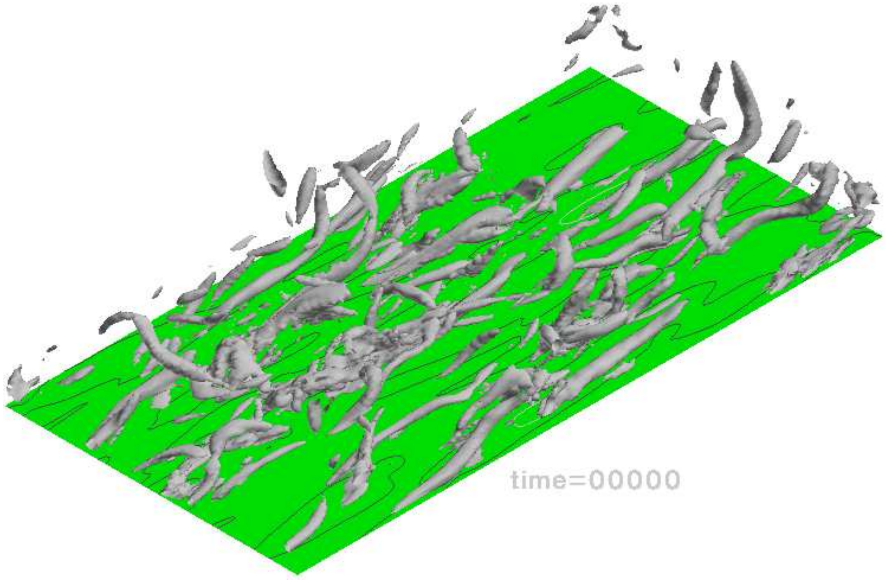

Figure 2: The coherent structures of near-wall turbulence.

With the abundance of computational power currently available, and the relatively mature numerical techniques to leverage this power, many complex flows of interest may now be modeled accurately by simulation of the Navier-Stokes equation or filtered versions thereof. Further, a significant expansion of this capability might be projected over the next several years, resulting from both increases in computational power (via parallel computers currently being developed) and improvements in numerical techniques. We must ask ourselves, however, what we plan to do with all of this simulation capability.

Leveraging these two advances—a mathematical understanding of the equation governing turbulence and the computational ability to simulate turbulence accurately—the time is ripe to make the next major advance: the control and optimization of flow systems dominated by the effects of turbulence for desired engineering objectives. Such a goal is now feasible because it is possible to utilize the equation governing turbulence in a computational setting to optimize and control a turbulent flow in an efficient manner. The objective of this work is to study and demonstrate how this may be done. This report has been organized as four self-contained articles. These articles lay out a progression over the following four topics:

I linear feedback for a linear flow system (transition),

II linear feedback for a nonlinear flow system (convection),

III predictive optimization of controls for a nonlinear flow system (turbulence), and

IV a framework for the optimization of robust controls for nonlinear flow systems.

These four articles are now summarized briefly.

Part I - Optimal and robust control and estimation
of linear paths to transition

It is generally inappropriate to treat multi-scale systems governed by nonlinear PDEs with classical control techniques (such as PID approaches and techniques based on Bode and root-locus analyses) or even "modern" control techniques (such as linear optimal control with Kalman filtering—designs based on analysis of transfer function norms and solved with Riccati-based approaches). Such techniques generally have been developed with low-order systems in mind, where the order of the state $N \lesssim O(100)$, whereas accurate simulations of turbulence routinely require $N \gtrsim O(10^6)$. Further, some of these techniques, such as approaches based on root-locus (eigenvalue) analysis, are not appropriate for systems with highly nonorthogonal eigenvectors, as often encountered in fluid mechanics. However, some specific PDE systems (under linear approximation) do decouple into several low-order systems amenable to analysis by the usual control strategies. Transition in a plane channel flow is just such a problem. This model problem, in addition to being a physically relevant problem in its own right, provides a very good starting point with which to introduce the topics of control and estimation in a fluid-mechanical setting. This problem also introduces the important concept of the noncooperative game framework, referred to as robust control, which is extended to the nonlinear Navier-Stokes equation in Part IV.

Part II - Linear control and estimation of nonlinear chaotic convection:
harnessing the butterfly effect

A control engineer's first instinct after solving the linear transition control problem of Part I might be to attempt to apply the linear control feedback so determined directly to the fully nonlinear problem of turbulence. This section examines the hazards of such a procedure (applying linear feedback to nonlinear systems) by examining a very low-order nonlinear convection problem which is easily amenable to analysis. The approach of applying linear feedback to both the controller and the estimator is ultimately made to work for this nonlinear convection problem, but only after the appropriate safeguards are taken. Further, the problem of turbulence is distinctly different from the convection problem studied here, as discussed in detail in this article. These differences appear to indicate that control of turbulence by simple linear feedback is a dubious proposition at best, and more appropriate feedback strategies need to be developed.

Part III - DNS-based predictive control of turbulence:
an optimal benchmark for feedback algorithms

For a mathematically rigorous framework for the control of nonlinear systems governed by the Navier-Stokes equation, we turn to the versatile yet cumbersome framework of predictive control. In this setting, a fully nonlinear flow problem (turbulent channel flow) is optimized over a receding finite horizon with the adjoint-based tools of optimal control theory. The results obtained (relaminarization of the turbulent flow with small amounts of blowing and suction) far surpass the results obtained with every other control algorithm tested in this flow to date.

The control strategy used is akin to that used to play a game of chess, in that future possible evolutions of the system are explored iteratively. The key feature that makes such an optimization tractable for the high-dimensional problem of turbulence, for which random or "brute-force" search algorithms are far too expensive even to consider, is that gradient information based on an adjoint analysis may be determined to update the controls efficiently for the desired purpose at each iteration. The adjoint field central to this approach is based on a mathematical statement of the engineering problem at hand and a nonlinear PDE representation of the equation governing the system of interest, obviating the need for *ad hoc* assumptions about the turbulence dynamics. Techniques proposed to extend the approach implemented here to a practical setting are discussed in detail.

Part IV - A general framework for robust control in fluid mechanics

For the application of control feedback in practical situations for which only limited, noisy measurements of the turbulent flow will be available, a degree of "robustness" will be very important in the control algorithm. The topic of robust control is a natural extension to that of optimal control. The analogy used to motivate this noncooperative-game approach is Murphy's law: *If a worst case disturbance can disrupt a controlled closed-loop system, it will.* Thus, in the robust framework, the controls are optimized in the presence of a finite component of that disturbance which is maximally detrimental to the control objective. Controls which are effective even in the presence of such a disturbance are also effective in the presence of a broad class of other disturbances.

Rather than a simple minimization problem, as encountered in optimal control, the robust control problem is an infinite-dimensional extrapolation of a saddle-point problem for the "best" control in the presence of the "worst" disturbance. The mathematical character of this saddle point problem is explored in detail in this article.

It is well known, even in the linear setting, that the performance of optimal control strategies is significantly degraded in noisy environments. For example, in the optimal linear setting (see Part I), disturbances are accounted for only in the estimator, not in the computation of the control feedback. This leads to the possibility for inappropriate control feedback arising from those modes of the system which can not be estimated accurately. Similar difficulties may be expected in the nonlinear setting. The robust framework naturally couples the control and estimation problems, accounting for the possible effects of disturbances when computing the control. For this reason, robust control of the nonlinear Navier-Stokes equation has been introduced and analyzed as a viable new tool for the optimization of practical, disturbed turbulent flow systems in which only limited, noisy measurement information is available. This robust approach is likely to be a key component of much of our future work in this area.

Literature surveys

The field of flow control is a very active field of research, and very significant contributions have been made by many different research groups. In order to review this field thoroughly, multiple comprehensive literature surveys have been completed in the course of this work. The first, published in June of 1994, is included as the appendix to this report. This review encapsulates the broad field of flow control at its state-of-the-art in 1994, citing scores of relevant references ranging from experimental, computational, and theoretical approaches to related flow control problems. It also summarizes our motivation for developing mathematically-based control algorithms for flow control problems, identifying a general taxonomy of schemes commonly used to attack flow control problems and describing how mathematically-based approaches fit with this big picture. In short, four general catagories of schemes are identified by examining the extent to which they are based on the governing flow equations. These catagories are:

- adaptive schemes,
- schemes based on understanding of dominant physics,
- schemes based on reduced-order systems, such as dynamical systems theory, and
- schemes based on optimal control theory.

It is noted that simple flow systems which behave approximately linearly may be controlled effectively by simple "black-box" strategies or by control strategies inspired by a general understanding of the flow physics. However, for distributed, multi-scale, chaotic flow systems such as turbulence, the effectiveness of such approaches is limited, and we are thus motivated to base our control algorithms on the Navier-Stokes equation known to govern the phenomena of interest.

In addition to this dedicated review paper, the literature was again carefully reviewed when composing each of the four main parts in the body of this report, and references which appeared in the last few years were paid particularly close attention. In this regard, note especially

a. the introduction and conclusions of Part II, which discuss feedback approaches for the control of low-dimensional nonlinear systems and their possible connection with the problem of control of turbulence, and

b. the introduction of Part III, which essentially re-reviews recent approaches taken to the turbulence control problem and discusses where optimal control theory fits into the larger picture.

Future work

Given current trends in simulation capability, it is clear that control and optimization of distributed systems by mathematically-based approaches will be commonplace in 20 years. The foundation for this design paradigm is being set today. The present collection of articles represents four new steps in this direction. Much work remains to be done. Several projects have been initiated which follow up on the present work and are currently under active investigation.

Acknowledgments

"When I heard the learn'd astronomer,
When the proofs, the figures, were ranged in columns
before me,
When I was shown the charts and diagrams, to add, divide,
and measure them,
When I sitting heard the astronomer where he lectured with
much applause in the lecture-room,
How soon unaccountable I became tired and sick,
Till rising and gliding out I wandered off by myself,
In the mystical moist night-air, and from time to time
Looked up in perfect silence at the stars."

- Walt Whitman (1865)

This thesis is dedicated to my parents, who taught me to keep looking up.

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