# OPTIMAL CONTROL OF TRANSITION INITIATED BY OBLIQUE WAVES IN CHANNEL FLOW

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## ABSTRACT

Using two different approaches to optimal control in channel flow an effort is made to try to identify differences and similarities. One approach is to use the Navier–Stokes equations and apply a gradient based optimization technique to find the optimal control. The other approach is to make use of the linearized equations known as the Orr–Sommerfeld–Squire equations to compute the optimal control. Limiting ourselves to look only at oblique wave perturbations we compare the resulting energy evolution from application of the respective control strategies. Qualitatively the performance of the two approaches are similar, at least when they work under comparable conditions. The non-linear control can be more aggressive initially since there is no direct limitation on the time derivative of the control even though the discretization implicitly enforces some degree of penalty. Adjusting the parameters properly we can show that the control from the two approaches are very similar. Also we try to quantify the performance of the estimator based control, or compensation, using only measurements on the wall, compared to the full-state information control. The performance of the compensator is found to be good for small perturbations, especially if a good initial guess can be provided.

## INTRODUCTION

The goal of this work is to develop methods to prevent transition to turbulence. We determine how to do control in the optimal way given the method of controlling the flow, and an objective function describing the features of the flow to be controlled. The method of actuation chosen here is blowing and suction at the walls, since it is a fairly simple way of acting on the flow, and also because it is a technique that is widely used. Blowing and suction has successfully been used for similar problems, namely control of turbulence, where complete relaminarization was obtained in Bewley *et al.*(1999). The blowing and suction is applied to flow in a channel, where we can find many of the interesting bypass transition scenarios. We use two different approaches to optimal control, one based on the non-linear Navier–Stokes equations and one on the 3D Orr–Sommerfeld–Squire equations.

In the non-linear case, we use the adjoint equation to compute objective function gradients. It is an efficient method in the sense that only two computations are required for each optimization iteration independent of the number of degrees of freedom of the control. First the state equation ( Navier–Stokes ) is solved and then this solution is used as input to the adjoint equation that is solved next and gives the gradient of the objective function. Optimization is performed with a limited memory quasi Newton method described in Byrd *et al.*(1994). The resulting control will be optimal for the specific perturbation and time domain studied.

In the linear case, optimal ($\mathcal{H}_2$) controllers and estimators are developed for the 3D Orr-Sommerfeld–Squire equations at a large array of wavenumber pairs $\{k_x, k_z\}$, using a technique closely related to that described by Bewley & Liu (1998) , and transformed to the physical domain. The feedback gains for both the control and estimation problems are shown to be represented by well-resolved, spatially-localized convolution kernels, see Högberg and Bewley (2001). The resulting control kernels represent the optimal feedback strategy for an arbitrary perturbation to minimize the energy over the infinite time domain. The physical-space controller, estimator, and compensator which combines them are then applied in (non-linear) direct numerical simulations of flow in a channel with oblique wave perturbations. The different transition scenarios in channel flow have been bench-marked by Reddy *et al.* (1998).

## CONTROL PROBLEM

An adjoint direct numerical simulation (DNS) code has been developed based on a existing spectral channel flow code by Lundbladh *et al.* (1992) to perform the non-linear as well as the linear control computations. Temporal DNS are performed. Fourier modes are used for the span-wise and stream-wise directions and Chebyshev collocation in the wall normal direction. The modification necessary to solve the adjoint equations involves a change in what corresponds to the non-linear terms for the Navier–Stokes solver to forcing terms depending on the choice of objective function. Solution of the adjoint equation requires full information about the solution of the Navier–Stokes equation in space and time. Based on previous findings we have used a discretization of the continuous equations instead of an exact discrete adjoint, see Högberg & Berggren (2001). For simulation with an estimator or compensator a similar code bench-marked by Bewley *et al.* (1999) with finite differences in the wall normal direction is used.

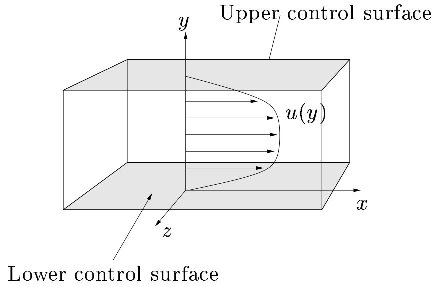


Figure 1: Geometry for control of channel flow.

The flow geometry is the one in figure 1 with blowing and suction applied at both walls of the channel and periodic boundary conditions on the stream-wise and span-wise directions. For the non-linear optimization the objective function $J_{nl}$ is,

$$J_{nl} = \frac{\varepsilon}{2}\int_{T_{C0}}^{T_{CF}} \varphi^2(x,z,t)\, d\Gamma + \frac{1}{2}\int_{T_{E0}}^{T_{EF}} (u(x,y,z,t) - u_{\text{lam}}(y))^2\, d\Omega$$

where the control $\varphi$ is applied from time $t = T_{C0}$ to $T_{CF}$ on the boundary $\Gamma$ and the energy of the deviation from the laminar profile $u_{\text{lam}}$ is measured from time $t = T_{E0}$ to $T_{EF}$ in the computational domain $\Omega$. The spatial resolution of the control is the same as for the simulation and temporally the control is linearly interpolated in time. In the linear case the objective function $J_l$ is,

$$J_l = \frac{\varepsilon}{2}\int_0^\infty \frac{\partial\varphi(x,z,t)}{\partial t}^2\, d\Gamma + \frac{1}{2}\int_0^\infty (u(x,y,z,t))^2\, d\Omega$$

and the controllers are computed by solving an optimal control problem for each wavenumber pair separately. In short a Riccati equation containing the Orr–Sommerfeld–Squire matrices, the energy measure matrix and the forcing matrix is solved to find the optimal controller. For further details see Högberg and Bewley (2001). The estimator used is an extended Kalman filter and is computed in a way similar to that of the linear controller. The objective function in this case measures the energy of the state error and of the forcing used. The penalty parameter for the forcing is denoted $\alpha$. A low value of alpha should be used when the measurements are expected to be free from noise and a high value for noisy measurement data. Notice that the linear controllers and estimators are computed off-line once and for all and then applied online in the simulations.

## SIMULATIONS, RESULTS AND DISCUSSION

All simulations are performed at $Re = U_c h/\nu = 2000$ where the Reynolds number is based on the half channel height $h$ and the centerline velocity $U_c$. The resolution is $8\times65\times8$ Fourier $\times$ Chebyshev $\times$ Fourier modes in $x \times y \times z$ respectively. For the code with finite differences in the wall normal direction 81 points are used. In all simulations the same particular oblique wave perturbation is used as initial condition at $t = 0$. Control is applied in all Fourier modes on both the upper and lower wall of the channel, and for the estimator case measurements are done in all Fourier modes at both walls. The control is parameterized in the non-linear case to a specific number of degrees of freedom with equispaced distance $\Delta t$ while in the linear case it is free to change arbitrarily at every time step. For the non-linear optimal control computations about 200 velocity fields are saved during the solution of the flow, and these are then linearly interpolated when used in the solution of the adjoint equations. The penalty parameter $\varepsilon$ is zero in the non-linear computations since the corresponding objective function for the linear controller does not add extra penalty on the control velocity.

### Non-linear control

We have computed linear controls and corresponding non-linear controls for comparison in terms of performance to investigate how close the optimal linear control is to the optimal non-linear control, with a similar objective function. To allow for comparison between the two different controls we need to make sure that the time interval is long enough to be considered as infinite by the non-linear controller. We also need to adjust the time resolution of the control to get a comparable penalty on the time-derivative. Even if we adjust the parameters to give similar objective functions the non-linear controller still has the advantage of being able to adjust to the particular perturbation an make use of non-linear effects. In

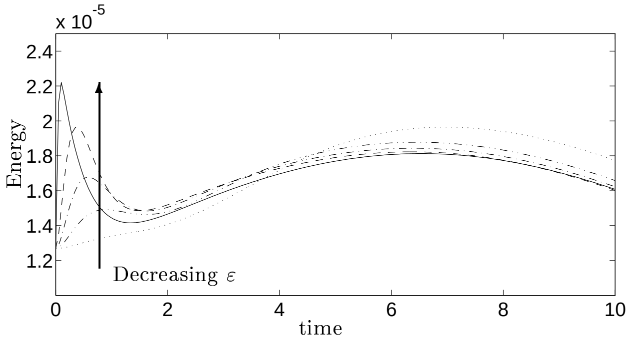


Figure 2: Energy evolution of controlled cases with linear control for different values of the penalty parameter $\varepsilon = 0.01, 0.05, 0.1, 0.2, 0.5$ as solid, dash, dash-dot, dash-dot-dot, dot respectively.

figure 2 the energy evolution of the perturbation is plotted for different values of the penalty on the time-derivative of the linear control. A similar restriction can be put on the non-linear controller by changing the time resolution of the control. In figure 3 the effect of changing this resolution for the non-linear controller is plotted. One can say that there is a qualitative correspondence between the penalty on the time-derivative in the linear case and the time-resolution of the control in the non-linear case. Notice that the resolution of the controller in time is not related to the time step in the simulations. Two cases, one with linear control and one with non-linear, with similar behavior initially are compared in terms of the energy evolution if figure 4. Except for the small difference initially it is hard to distinguish one curve from the other. It seems as the linear controller does an almost as good job as the non-linear one in this case. Evaluating the objective functions gives a 2% higher value for the linear controller in this case.

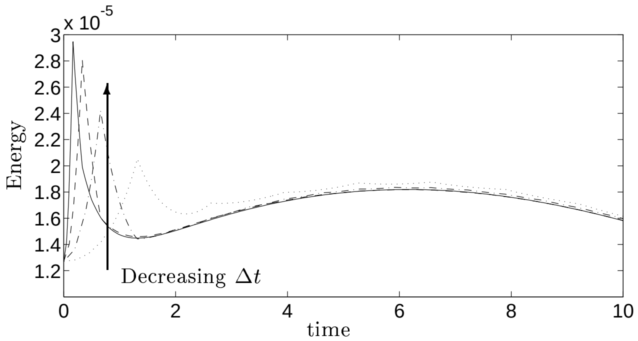


Figure 3: Energy evolution of optimally controlled cases with non-linear control for different time resolution of the control. Using 300,150,75,37 degrees of freedom in time as solid, dash, dash-dot, dot respectively.

### Linear control

In the linear case we have pre-computed convolution kernels that are applied online in

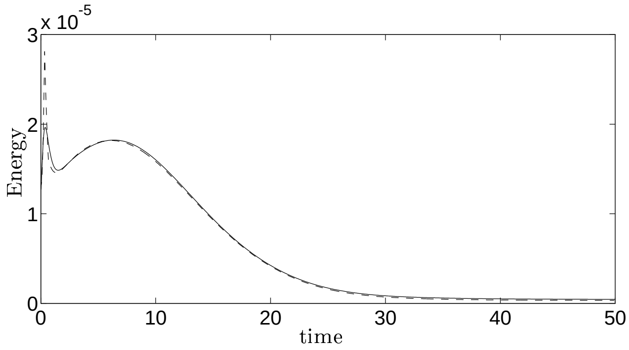


Figure 4: Energy evolution of optimal linear control case with $\varepsilon = 0.05$ ( solid ) and optimal non-linear control with 150 degrees of freedom in time ( dash ).

the DNS. The controller, utilizing full information of the flow-field, can prevent transition at perturbation levels well above the uncontrolled transition thresholds computed by Reddy *et al.*(1998), see Högberg and Bewley (2001). The estimator converges exponentially to the correct state of the flow as shown in figure 5. Unfortunately the rate of this convergence is somewhat low, and there was no way of speeding it up further using the present formulation of the estimation problem. In figure 6 the effect of changing the penalty parameter $\alpha$ is shown. It turns out that it is favorable to decrease it to obtain speedup of the convergence, but only up to a certain limit. Estimation of the oblique wave perturbation with $\alpha = 0.01$ is illustrated in figure 7. There is a time lag in the energy evolution of the estimator compared to the true state, but eventually the estimator gets closer and closer.

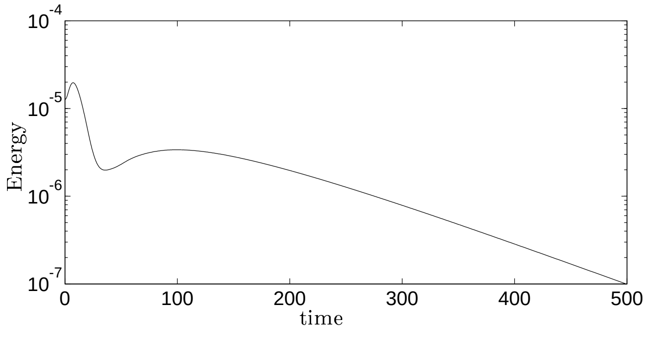


Figure 5: Energy of state error in estimation for $\alpha = 0.01$.

Combining the estimator and controller into a compensator where the flow is controlled based only on wall measurements is the next step. With perfect initial data for the estimator the performance would be the same as for full information control. Starting with a unperturbed flow in the estimator is more of a challenge, and the result from this is plotted in figure 8. The compensator is able to lower the energy growth substantially but not as much

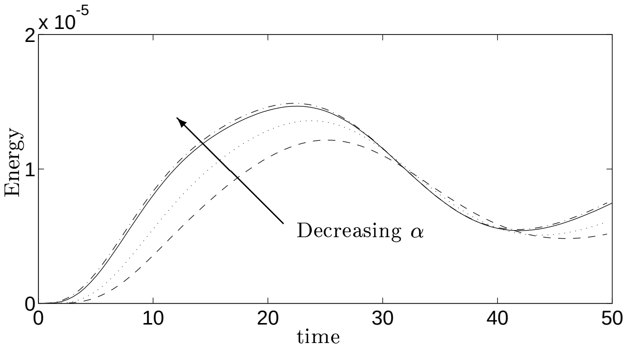


Figure 6: Energy evolution in estimator with different values on the penalty parameter $\alpha$= 0.001, 0.01, 0.05, 0.1, as dash-dot, solid, dotted, dashed respectively.

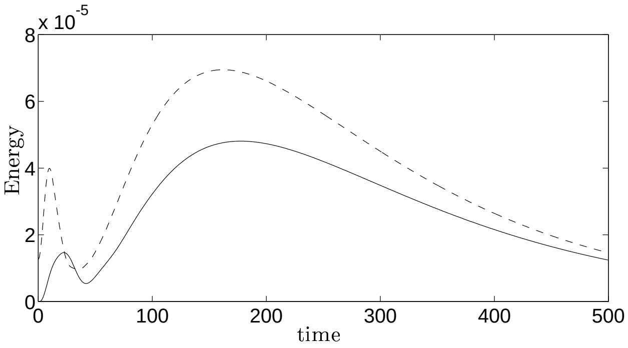


Figure 7: Energy evolution of the estimator with $\alpha = 0.01$ (solid ) and the true uncontrolled state ( dash ).

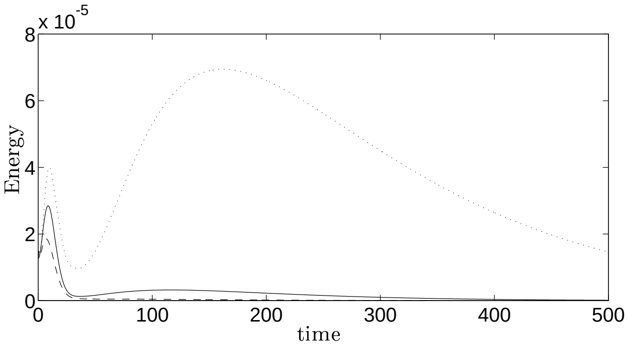


Figure 8: Full information linear controller ( dashed ) and compensator ( solid ) performance compared to the uncontrolled ( dotted ) energy evolution.

as the full information controller. In a spatial case one could imagine having the estimator upstream of the controller. Here that would correspond to giving the estimator a head start before applying control. Estimating the flow until $t = 50$ and then applying the compensator and comparing it to the full information controller applied at $t = 50$ shows that the compensator performance is now close to that of the full information controller. In figure 9 the energy evolution for the full information control case is compared to that of the "head started" compensator and the regular compensator with zero perturbation as initial guess at $t = 50$.

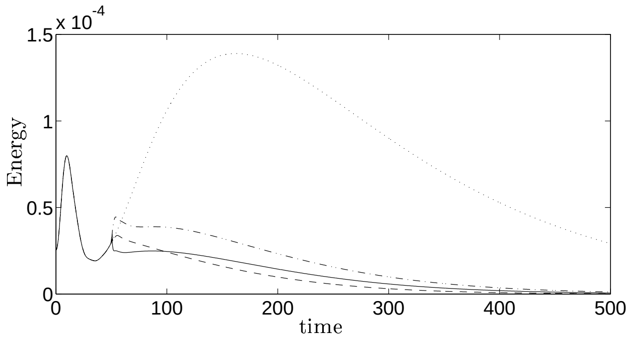


Figure 9: Full information controller ( dash ) and "head started" compensator ( solid ) and regular compensator (dash-dot ) performance compared to the uncontrolled ( dot) energy evolution.

## Conclusions

The action of the optimal linear controller is very similar to that of the optimal nonlinear control. A comparison with a nonlinear optimal controller, based on iterative adjoint computations, shows only small differences to the controllers based on the linearized equations. The perturbation evolution can be reproduced from wall measurements online, using an estimator with exponential convergence rate after some initial transients. When basing the control on wall measurements only the performance is not as good, but still energy growth is reduced. Giving the compensator a better initial guess improves the performance substantially. One future focus for linear compensation should be development of better estimators with fast convergence.

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